Solutions
Problem Set 2

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1. For Bob, we are given that $MRS_{P-B} = 2$

a) $MRS_{P-B} = -\frac{\Delta B}{\Delta P} = 2 \Rightarrow \Delta B = -2\Delta P$

Let $\Delta B = 1$, then $\Delta P = -\frac{1}{2}$

Hence, Bob would give up 1/2 pizza for 1 additional beer.

b) Let $\Delta P = 1$, then $\Delta B = -2$. Hence Bob would give up 2 beers for 1 additional pizza.

c) For Bob $MRS_{P-B} = 2$; and for Carol $MRS_{P-B} = 4$

This implies that Bob is willing to give up 2 beers for 1 additional pizza, but Carol is willing to give up 4 beers for 1 additional pizza. Because $4 > 2$, Carol must value pizza more than Bob. [Note: Similar reasoning implies that Bob values Beer more than Carol].

2. Write down equations for the utility function and indifference map for each of the cases given.

a) The MRS_{P-B} or MRS_{B-P} is constant. This implies the utility function is linear in Beers (B) and Pizza (P). That is, we have a case of perfect substitutes.

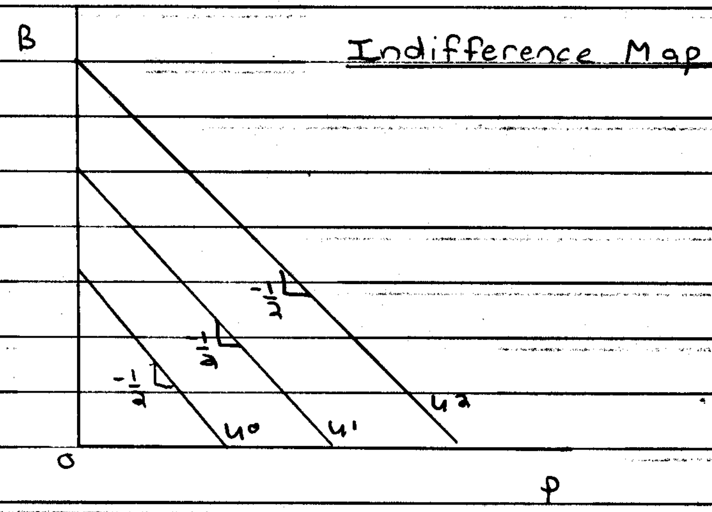
(1) $U = 2B + 1P$ or, more generally, [Perfect Substitutes]

(1') $U = \alpha B + \gamma P$, where $\alpha = 2\gamma$.

Notice that we can write (1) in the following form:

(2) $B = \frac{U}{2} - \frac{1}{2}P$

The coefficient of $-\frac{1}{2}$ on P implies that Carol is willing to give up 1/2 Beer for 1 additional pizza, or 1 Beer for 2 additional pizzas



b) This is a case of perfect complements because Bob consumes Beer and Pizza in fixed proportions: 2 Beers with every pizza. His utility function is given by

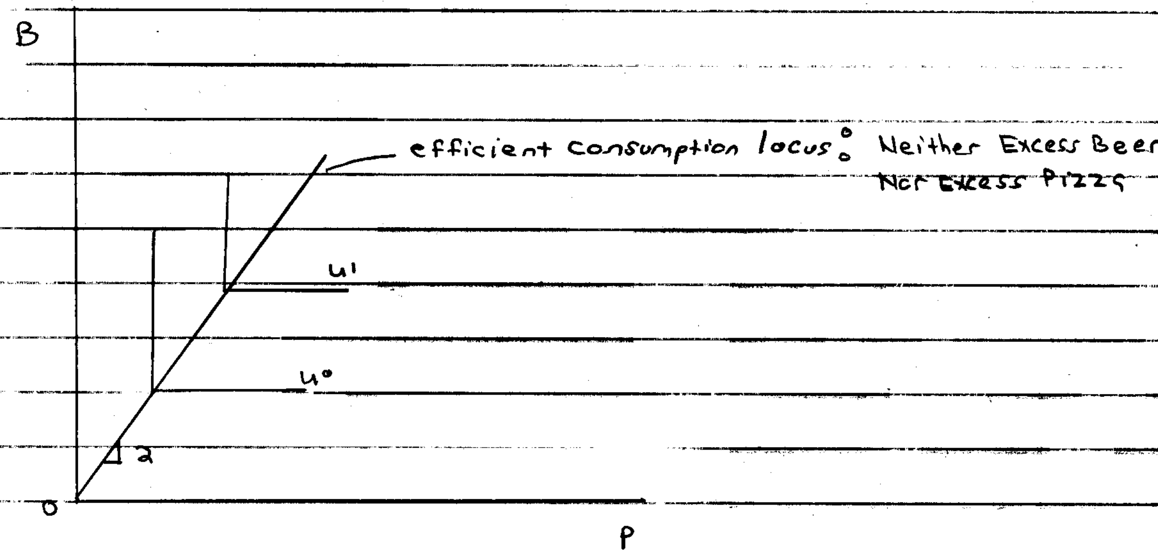
(3) $U = \beta \text{ Min}\{\frac{1}{2}B, P\}$ β is an unknown.

Set $\frac{1}{2}B = P$ and obtain $B = 2P$; that is for every pizza Bob consumes, he also consumes 2 Beers. We are also told that when $P = 4$ and $B = 10$, $U = 32$. Hence

(4) $32 = \beta \text{ Min}\{5, 4\} = \beta \cdot 4 \Rightarrow \beta = 8$. Hence,

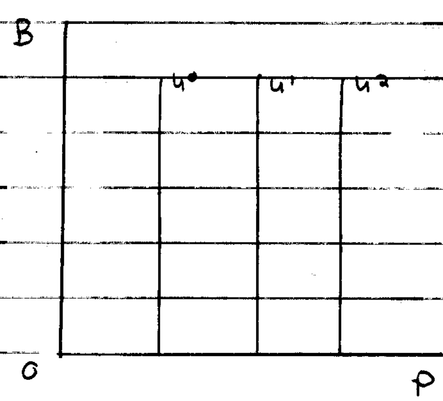
Bob's utility function is given by

(5) $U = 8 \text{ Min} \{ \frac{1}{2} B, P \}$



c) We are told that Kathy loves pizza and is neutral toward Beer. This means that Kathy derives no positive (or negative) utility from Beer. Also, she derives 4 units of satisfaction for each pizza she consumes [Note: This value of 4 is a constant]

(6) $U = 4P$



3 Given $P_p = \$4$, $P_B = \$2$ and $I = \$100$

a) Determine equilibrium number of Beers and Pizza?

(1) $U = 2P + 4B$ or $B = \frac{U}{4} - \frac{1}{2}P$

Budget Constraint: $2B + 4P = 100$ or $\boxed{B = 50 - 2P}$

Observe that the slope of the budget constraint (-2) and the slope of indifference curve ($-\frac{1}{2}$) are not equal. Hence, we will have a corner solution (i.e., allocate entirety of income to Beers or entirety of income to Pizzas) We need to determine which outcome generates higher utility.

If Purchase only Pizzas, $P = \frac{100}{4} = 25$

and $U = 2(25) = 50$

If Purchase only Beers, $B = \frac{100}{2} = 50$

and $U = 4(50) = 200$.

Since $200 > 50$, Purchase only Beers.

Equilibrium Outcome: $\boxed{B^* = 50;\ P^* = 0;\ U^* = 200.}$

(2) $U = 2\min\{P, \frac{1}{3}B\}$ Solve following 2 equations simultaneously:

$B = 3P$ (Efficient Consumption Locus)

$B = 50 - 2P$ (Budget Constraint)

$3P = 50 - 2P \Rightarrow 5P = 50$ and $P = 10$

Revised

When $P = 10$, $B = 30$ and $U = 2\min\{10, 10\} = 20$

Hence, Equilibrium Outcome: $\boxed{B^* = 30,\ P^* = 10,\ U^* = 20.}$

(3) $U = 4BP$ (Given $MU_B = 4P$ and $MU_P = 4B$)

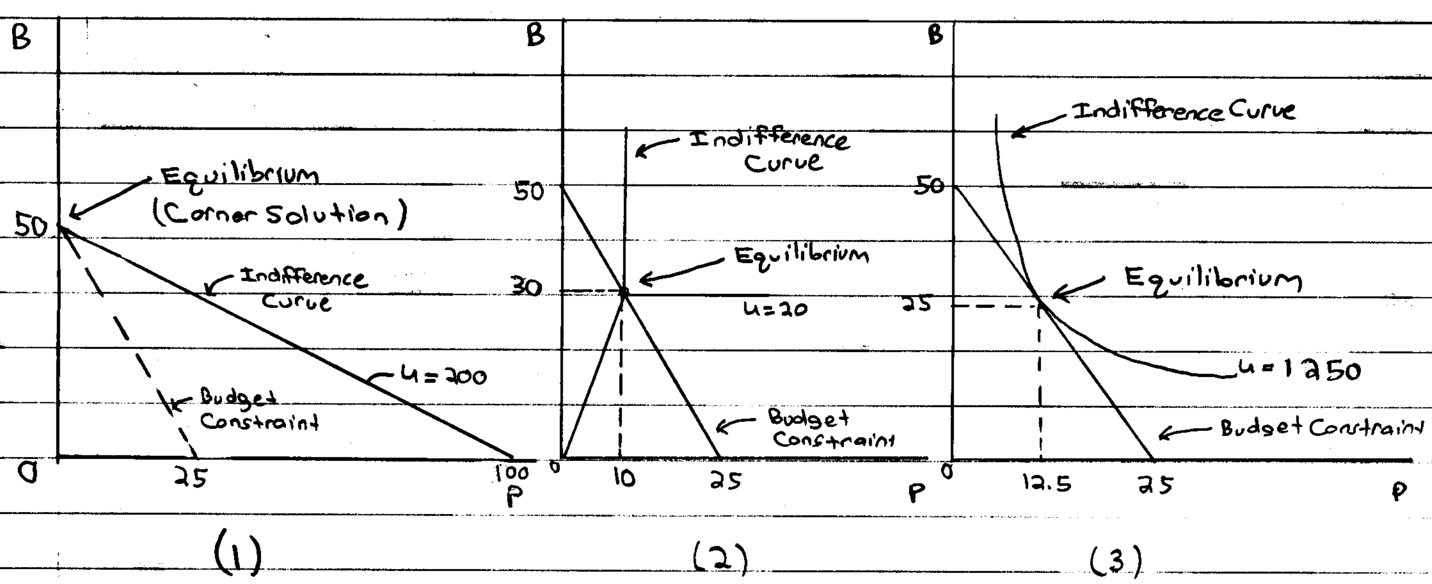
Consumer Equilibrium (Interior Solution) Requires That

$$\frac{MU_B}{P_B} = \frac{MU_P}{P_P} \Rightarrow \frac{4P}{2} = \frac{4B}{4} \Rightarrow B = 2P$$

Also, the budget constraint must be satisfied. Hence, $B = 50 - 2P$. Solving these 2 equations simultaneously yields $4P = 50$, or $P = 12.5$ and $B = 25$. This consumption bundle generates utility of $U = 4(25)(12.5) = 1250$. Hence, the equilibrium outcome is given by:

Equilibrium Outcome: $\boxed{B^* = 25;\ P^* = 12.5;\ U^* = 1250.}$

b) Graphical illustration of Results



4. Compute marginal rates of substitution for each of the utility functions in question 3 when $P = 10$ and $B = 20$.

(1) $u = 2P + 4B$ and $B = \frac{u}{4} - \frac{1}{2}P$ =>

$MRS_{P-B} = \frac{1}{2}$; $MRS_{B-P} = 2$ [These are constants]

(2) $u = 2 \min\{P, \frac{1}{3}B\}$

Efficient consumption requires $B = 3P$. At $P = 10$ and $B = 20$, we have too many pizzas and not enough beers. Hence

$MRS_{P-B} = 0$ and $MRS_{B-P} = \infty$

(3) $u = 4BP$ [Recall: $MU_B = 4P$ and $MU_P = 4B$]

$$MRS_{P-B} = \frac{MU_P}{MU_B} = \frac{4B}{4P} = \frac{B}{P} = \frac{20}{10} = 2$$

$$MRS_{B-P} = 1/MRS_{P-B} = \frac{1}{2}$$

5. Utility Functions From Question 3 On Problem Set 2.

(1) $U = 2P + 4B$; (2) $U = 2\min\{P, \tfrac{1}{3}B\}$; (3) $U = 4B \cdot P$

1. Utility Function (1) is perfect substitutes. Three possibilities For Consumer Equilibrium. [Normalize $P_P = 1$]

Demand Functions

$$P = \begin{cases} \dfrac{I}{P_P}, & P_B > 2P_P \\ \dfrac{I - P_B \cdot B}{P_P}, & P_B = 2P_P \\ 0 & P_B < 2P_P \end{cases} \qquad B = \begin{cases} 0, & P_B > 2P_P \\ \dfrac{I - P_P \cdot P}{P_B}, & P_B = 2P_P \\ \dfrac{I}{P_B}, & P_B < 2P_P \end{cases}$$

(2) Perfect Complements. The efficient consumption locus is given by (i) $B = 3P$; The budget Constraint is given by (ii) $B = I/P_B - P_P/P_B \cdot P$. Solve (i) and (ii) simultaneously

$$3P = \frac{I}{P_B} - \frac{P_P}{P_B} \cdot P \Rightarrow P\left[3 + \frac{P_P}{P_B}\right] = \frac{I}{P_B} = P[3P_B + P_P] = I$$

or

$$\boxed{P = \frac{I}{[3P_B + P_P]} \quad \text{and} \quad B = \frac{3I}{[3P_B + P_P]}}$$ Demand Functions

(3) "Typical Preferences" [Given $MU_B = 4P$; $MU_P = 4B$]

Equilibrium Condition : $\dfrac{MU_P}{P_P} = \dfrac{MU_B}{P_B} \Rightarrow \dfrac{4B}{P_P} = \dfrac{4P}{P_B} \Rightarrow \boxed{B = \dfrac{P_P}{P_B} \cdot P}$

Budget Constraint : $\boxed{B = \dfrac{I}{P_B} - \dfrac{P_P}{P_B} \cdot P}$

Solve Simultaneously

$$\frac{P_P}{P_B} \cdot P = \frac{I}{P_B} - \frac{P_P}{P_B} \cdot P \Rightarrow P[2P_P] = I \Rightarrow$$

$$\boxed{P = \frac{I}{2P_P} \quad \text{and} \quad B = \frac{I}{2P_B}}$$ Demand Functions