Returns to Academic Standards and the Selectivity of Colleges

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Abstract

The paper develops a model of returns to college education that reflects the following stylized facts which characterize the recent changes in the American higher education: explosive growth of the overall enrollments combined with diverging selectivity of US colleges; rapid growth of wage premium in the top deciles, and its stagnation in the lower deciles of the wage distribution among college graduates. The paper focuses on the relationship between the choice of curricular standards by colleges and the distribution of returns to students in terms of human capital outcomes and, accordingly, their college premia. We argue that a student's college value added is a non-linear function of the relationship between student's prior preparation and the curricular standard chosen by the college. Thus colleges affect the distribution of outcomes in the student population by choosing curricular standards in accordance with their objectives.

We apply the model to analyze the competition between more selective colleges, whose aim is to add the maximum aggregate value to the human capital of its student body (which can potentially compel a proportional sense of obligation in future alumni), with the less selective ones that in addition to the quality indicators also place explicit value on ensuring access to a larger quantity of students. The competition amounts to the choice of the optimal location by the colleges along the axis of student ability. We show that as a result of this intercollegiate competition, the rise in college premium will cause less selective colleges to lower their standards further, while the effect on more selective colleges is the opposite. The resulting distribution of human capital attainment will feature gains at opposite ends of the ability distribution with stagnation and even relative decline in the middle.

1. Introduction

The evolution of the American higher education since the 1960-s has been characterized by the following set of stylized facts: explosive growth of the overall enrollments in post-secondary degree-granting institutions combined with diverging selectivity of US colleges, rapid growth of the average college wage premium since about 1980 accompanied, since mid-1990-s, by growing wage inequality among college graduates (with persistent growth of college premium in the top deciles and its stagnation in the lower deciles of its distribution).

The first set of facts concerning rapid growth of enrollments and overall student access to public colleges, and a more tepid one in private not-for-profit colleges is presented in Tables 1 and 2 (see also Kaganovich and Su, 2013).

Hoxby (2009) documents clear trends in increasing selectivity of historically more selective colleges, i.e., more perfect sorting (better "fanning out" as per Hoxby, 2009) of students, with respect to their pre-college preparation, across the distribution of colleges ranked by their historical selectivity. The data also show that the students "sorted" into the most

Table 1: Enrollment in post-secondary degree-granting institutions, in thousands (Digest of Education statistics, 2009).

Year	Public	Private not-for-profit
1959	2,181	n/a
1969	5,897	2,088
1979	9,037	2,461
1989	10,578	2,731
1999	11,309	3,052
2008	13,972	3,662

Table 2: Enrollment in post-secondary education by age group, in per cent (Digest of Education statistics, 2009).

Year	18-19 year-olds	20-24 year-olds	25-29 year-olds	30-34 year-olds
1959	n/a	12.7	4.9*	2.4*
1969	n/a	23.0	7.9	4.8
1979	34.6	21.7	9.6	6.4
1989	41.6	27.0	9.3	5.7
1999	44.1	32.8	11.1	6.2
2008	48.6	36.9	13.2	7.3

*Data for 1959 unavailable; reported for 1960.

selective schools are getting an increasingly better deal: while their tuition rises, the overall resources spent on them grow even faster, which translates into increasingly superior college premium these students will receive. While better sorting increases selectivity at the top, it has the opposite effect at the bottom where selectivity and by implication academic standards, decline. The latter fact correlates with the stagnation of college premium at the lower end of the distribution of college graduates, as documented by Autor et al. (2008).

This paper is aimed at developing a model of returns to college education that would account for the above stylized facts, particularly as it relates to the diverging selectivity of colleges. We focus on the relationship between the choice of curricular standards by colleges and the distribution of returns to students in terms of human capital outcomes and, accordingly, their college premium.

The literature on sorting in higher education largely owes to the seminal paper by Rothschild and White (1995) (see also its discussion by Winston, 1999) who while not specifically considering academic standards of colleges, do posit that quality of students largely determines the quality and reputation of colleges and thus students constitute inputs in production of human capital by the colleges. Their model, further developed by Epple et al. (2002, 2006), develops a theory of the quality differentiation of colleges where schools, ranked by their selectivity, competitively attract relevant segments of student population using merit based financial aid and tuition to ensure appropriate levels of human and financial factors of their operation.

While most of the literature devoted to the production of human capital focuses on such inputs in this activity as students' ability and school as well as parental resources, Costrell (1994) and Betts (1998) pioneered the exploration of the role of academic standards on student sorting, effort, and wage premium. They underscored the two-fold role of the standards: (i) signaling the average productivity level of college graduates to employers who are uninformed about individual characteristics of the graduates, and (ii) the enforcement of student effort required for attaining the degree and thus enhancing its quality. Thus as educational standard is raised in Costrell-Betts model, educational achievement will increase among all college graduates, and so will their wage. The group that will be affected negatively by the change includes those who lose access to higher education due to their failure or unwillingness to meet higher standards.

Production functions of college education in Su (2004) and Gilpin and Kaganovich (2012) feature academic standard as a threshold pre-college preparation, such that only students exceeding the standard can benefit from higher education. In the former paper, the idea of academic standard is used to underscore the importance of investment in basic, pre-college education for adequate returns at both stages. In the latter paper, the presence of the curricular threshold introduces non-linearity in returns to college education and thereby inequality in college premium disproportionately favoring students with superior pre-college preparation.

In this paper, we further focus on curricular standards as parameters of education production function and develop a model of quality differentiation among colleges in terms of these standards. The central argument that we advance in the model, based in part on educational psychology literature (see, in particular, van Geert, 1998, and van Geert and Steenbeek, 2005), is that a student's college value added is a non-linear function of the relationship between student's prior preparation and the curricular standard chosen by the college.[1] Thus colleges affect the distribution of outcomes in the student population by choosing

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[1] The concept of an active role played by curricular benchmarks set by an instructor for student's educational achievement was introduced by one of the founders of educational psychology Lev Vygotsky, who argued that instruction leads a learner into his/her *zone of proximal development (ZPD):* the distance between the actual prior developmental level as determined by independent problem solving and the level of potential development as determined through guided problem solving. In the ZPD, students cannot complete tasks unaided, but can do so with guidance, or "scaffolding" per Vygotsky. Jean Piaget, another pioneer of educational psychology, underscored the inherent biological foundations of human learning. However, his concept of cognitive conflict as an engine of learning (whereby the *adaptation* to the cognitive conflict, induced by the impulse to attain cognitive balance, is achieved through *assimilation* and *accommodation*) has lead for a now commonly accepted Vygotsky-Piaget synthesis. According to it, learning success requires that (i) the student can relate (assimilate) new information to that already known, i.e., it should be not too distant from it but "proximal" enough for building on the prior knowledge; and (ii) the new information must represent a sufficient challenge (i.e., it should not be too close to the base either) so it can ignite a cognitive conflict and hence the adaptive mechanism of cognitive development. This implies, as posited above, that a student's learning gain is a non-monotone, inverted U-shaped function of the level of challenge he/she faces: it first rises and then starts declining as the challenge faced by a student rises beyond an individually optimal level.

curricular standards in accordance with their objectives.

We apply the model to analyze the competition between two colleges. One is more selective: it values aggregate quality (human capital) which it adds to the pre-existing human capital of its student body (which can potentially compel a proportional sense of obligation in future alumni, as discussed in detail by Hoxby, 2012). The second college is less selective and in addition to the quality indicators it also places explicit value on ensuring access to a larger quantity of students. The competition between the colleges amounts to the choice of the optimal location by the colleges along the axis of student ability. We show that as a result of this intercollegiate competition, the rise in college premium will cause the less selective college to lower its standards further, while the effect on the more selective college is the opposite. The resulting distribution of human capital attainment will feature gains at opposite ends of the ability distribution with stagnation and even relative decline in the middle.

2. The Model

Students

Pre-college population is represented by the interval [0, 1]. Each individual ω is characterized by pre-college preparation (or ability) level $q(\omega)$. Since we do not model the process by which pre-college preparation is acquired, we will treat it as exogenously given and therefore synonymous with students' ability. We will therefore use the two terms interchangeably.

For simplicity, we impose

Assumption 1. $q(\omega)$ is distributed uniformly between 0 and 1.

This assumption will be of great help in terms of model's analytical tractability, but will make it harder to subject its results to a rigorous empirical hurdle.

Assumption 2. An individual deciding against college education will work for the wage $1+q(\omega)$ such that the benchmark level 1 is wage earned by the least able person.

Colleges

There are two colleges i = 1, 2. College 1 is selective (most typically though not necessarily private) and oriented toward high achievement (quality). College 2 is public and less selective, i.e., it is the "people's university" and balances quality along with "access", i.e., the quantity of students.

Assumption 3. The income of individuals graduating from college i is proportionate to the attained human capital: $wh_i(\omega)$, thus w is the college premium per unit of human capital against the wage rate of a worker without college education postulated in Assumption 2.

Curricular standards and learning

Human capital attainment of student ω in college i is given by:

$$h_i\,(\omega) = w^{-1}q(\omega) + A(b\bar{h}_i - q(\omega))(q(\omega) - \bar{h}_i) \qquad (1)$$

where $\bar{h}_i$ is the *lower benchmark* of the curriculum while $b\bar{h}_i$ with b >1 is the *curricular target* level of challenge; coefficient w^{-1} represents the (analytically convenient) in-college depreciation of pre-college human capital, while the scaling coefficient $A > 0$.

The above model corresponds to the one-iteration version of the model of "dynamic scaffolding" developed by van Geert and Steenbeek (2005). Their model describes the dynamics of interaction between a teacher and a student, akin to a game of pursuit and based on the Vygotskian theories of cognitive "proximal development", where a teacher sets optimal goals (those ensuring maximum progress) for a student based on his/her current level and then iteratively upgrades them as the progress is made (see footnote 1 in the Introduction to a brief outline of the Vygotsky-Piaget synthesis which serves as a common core of the modern educational psychology literature).

Our curricular target $b\bar{h}_i$ in (1) corresponds to the developmental goal in van Geert and Steenbeek (2005), while the coefficient b is the "optimal distance", i.e., the width, in multiplicative terms, of the zone of proximal development which characterizes in their model the level of challenge to a student which will maximize his/her educational gains.

Note that the human capital attainment function (1) is concave in each variable, $q(\omega)$ and $\bar{h}_i$. When the term $(b\bar{h}_i - q(\omega))$ is small, so the student's prior level of preparation $q(\omega)$ is close to $b\bar{h}_i$, the challenge is too small for much learning to occur; on the other hand, if $(q(\omega) - \bar{h}_i)$ is small (and certainly if it is negative), this means that the curricular target $b\bar{h}_i$ is too far away from the student's preparation $q(\omega)$, i.e., the latter is too low relative to the target for a substantial human capital gain to take place for this student under this curriculum standard.

A major feature of our analysis, which indeed reflects the realities of education systems under investigation, is that the curricular standard offered by each college is not tailored individually to students, but is applied to a diverse group of students, thus it will be inevitably sub-optimally low or high for individual students. It is easy to derive that the *student gaining the highest value added in college* i is the one with the ability level

$$\hat{q}_i = \min\{(1+b)\overline{h}_i / 2,\ 1\} \qquad (2)$$

which, in geometrical terms, merely states that the parabola $A(b\overline{h}_i - q)(q - \overline{h}_i)$ representing the value added component in (1) for a given value of $\overline{h}_i$ reaches its maximum at $q = (1+b)\overline{h}_i / 2$.

Reciprocally, we define $\overline{h}(q)$ as the *optimal curricular lower benchmark for a student of ability* q, i.e., the one that would yield for this student the highest educational value added. It is clear, according to the human capital production function (1), that

$$\overline{h}(q) = \frac{b+1}{2b} q \qquad (3)$$

for any $q \in [0, 1]$.

A comparison of relationships (2) and (3) leads to an important conclusion: the optimal curricular lower benchmark $\overline{h}(q')$ for student of ability q' defines the value added parabola $A(b\overline{h}(q') - q)(q - \overline{h}(q'))$ whose maximum is reached at point $q'' = (b+1)\overline{h}(q')/2 > q'$, i.e., to the right of the ability level q' (the more so, i.e., the further to the right, the higher the value of b). Thus the student of ability q', for whom curricular benchmark $\overline{h}(q')$ is optimal, <u>is not the greatest beneficiary of this curriculum</u>. Put differently, for any given student, his optimal curriculum is not the one, but rather <u>more challenging</u> than the one that yields him the highest value added among all students. Applied to the most able student in the population for whom q = 1, this implies, in particular, that his preferred curricular standard $\overline{h}(1)$ defines the value added parabola which reaches maximum outside of the range of the ability distribution.

College objectives and student sorting

According to the earlier outline, the two colleges are characterized by the following distinct objectives.

College 1, the "high-end" selective school, chooses its curricular standard $\overline{h}_1$ so as to maximize the aggregate value added for its students (which would be consistent, as discussed by Hoxby (2012), with the expectation that alumni donations will be proportionate to their gain derived from college education):

$$\max_{\overline{h}_1} F_1(\overline{h}_1) = \int_{\overline{q}_1}^{1} (b\overline{h}_1 - q(\omega))(q(\omega) - \overline{h}_1) d\omega \tag{4}$$

where the lower ability cut-off $\overline{q}_1$ is determined by student's individual indifference between attending college 1 or 2 based on the equalization of the value added (which of course must also be non-negative):

$$(b\overline{h}_1 - q(\omega))(q(\omega) - \overline{h}_1) = (b\overline{h}_2 - q(\omega))(q(\omega) - \overline{h}_2) \tag{5}$$

which implies

$$\overline{q}_1 = \frac{b}{b+1}(\overline{h}_1 + \overline{h}_2) \tag{6}$$

We note immediately that since this borderline student will only go to a university if it adds value to his human capital, formula (1) requires that $\overline{q}_1 \geq \overline{h}_1$. Combined with (6) this implies the following condition:

$$\overline{h}_1 \leq b\overline{h}_2 \tag{7}$$

Applying formula (3) for the optimal curricular lower benchmark to the student of the highest ability $q = 1$, we obtain $\bar{h}(1) = \frac{b+1}{2b}$. Moreover, we observe that the value added of each individual student in college 1

$$f(q, \bar{h}_1) = (b\bar{h}_1 - q)(q - \bar{h}_1) = -q^2 + q(1+b)\bar{h}_1 - b\bar{h}_1^2 \tag{8}$$

decreases as a function of curricular lower benchmark $\bar{h}_1$ when, according to formula (3). Since the objective (4) of college 1 is to maximize the aggregate value added of its students, it is clear that it would be suboptimal to set $\bar{h}_1$ at or above the level $\bar{h}(1) = \frac{b+1}{2b}$ preferred by the most able student. Thus we obtain the following helpful estimate:

$$\bar{h}_1 \leq \frac{b+1}{2b} \tag{9}$$

College 2, the less selective "people's university", in addition to focusing on the aggregate value added of its students also factors in college access to as many students as possible among its objectives. The civic stand on this principle may well correlate with an ulterior motive, given the political and public finance pressures on public colleges that encourage accessibility.

Specifically, it chooses its curricular standard $\bar{h}_2$ so as to maximize the following composite objective function:

$$\max_{\bar{h}_2} F_2(\bar{h}_2) = \int_{\bar{q}_2}^{\bar{q}_1} (b\bar{h}_2 - q(\omega))(q(\omega) - \bar{h}_2)d\omega \ + \gamma(\bar{q}_1 - \bar{q}_2) \tag{10}$$

where the lower ability cut-off $\bar{q}_2$ is determined by student's individual indifference between attending college 2 or none at all:

$$Aw(b\overline{h}_2 - q(\omega))(q(\omega)-\overline{h}_2)=1 \text{ or equivalently } q^2 - q(1+b)\overline{h}_2 + b\overline{h}_2^2 + (Aw)^{-1} = 0 \qquad (11)$$

while parameter γ signifies the weight given by the college to the goal of enhancing student access.

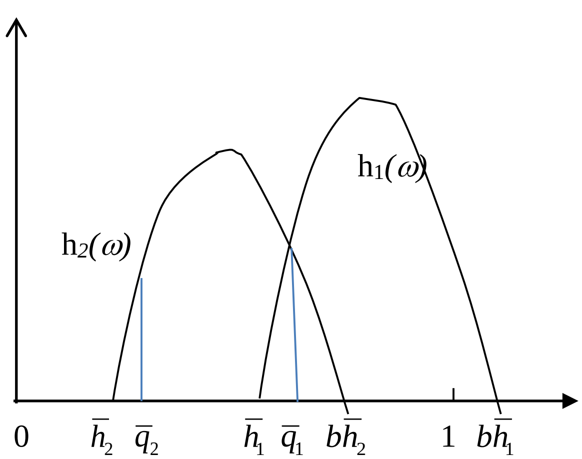
Equation (11) yields unique feasible solution

$$\overline{q}_2 = 2^{-1}\left[(b+1)\overline{h}_2 - \sqrt{(b-1)^2\overline{h}_2^{\,2} - 4(Aw)^{-1}}\right] \qquad (12)$$

since according to (2) it is clear that $\overline{q}_2 < (b+1)\overline{h}_2/2$ must be true. According to (12) this solution has the properties summarized in the following Lemma.

Lemma 1. $\overline{h}_2 \le \overline{q}_2 \le b\overline{h}_2$; $\frac{\partial \overline{q}_2}{\partial w} \le 0$ *where the derivative is taken directly as a function of* w.

The partition of the population in terms of college attendance decisions and the human capital gains attained in college is illustrated by the following figure.



Remark 1. The above analysis ensures, subject to condition (6), that all individual of ability higher than $\bar{q}_2$ will gain financially from attending a college, it is natural to also expect that all but the least able of them should gain in terms of human capital, i.e., that

$$h_{i(w)}(\omega) \geq q(\omega)$$

where $i(\omega)$ is the college chosen by student ω. The parametric conditions under which this property is satisfied will be worth exploring.

3. Results

We assume that the colleges' decisions about their curricular standards fully determine students' enrollment in them based on their individual maximization of human capital outcomes given those standards. Thus the colleges compete against each other for students by choosing their location on the axis of student ability (preparation). We define Nash equilibrium of this interaction and prove that it exists.

Lemma 2. *There is a value $\gamma_0 > 0$ such that under all $\gamma \geq \gamma_0$ (college 2 giving sufficiently high priority to enrollment) the Nash equilibrium exists and is unique.*

Next, we analyze the effect of an exogenous rise in the value of college premium $\boldsymbol{w}$ on the Nash equilibrium outcome. This analysis yields the main result of the paper.

Theorem. *Subject to the condition $\gamma \geq \gamma_0$, the rise in college premium w leads to the following effects:*

- *college 2 will relax its curricular standard, i.e.,* $\frac{\partial \overline{h}_2}{\partial w} < 0$ *,and will compel less prepared students to enroll:* $\frac{\partial \overline{q}_2}{\partial w} < 0$ *;*

- *college 1 will raise its curricular standard, i.e.,* $\frac{\partial \overline{h}_1}{\partial w} > 0$*, but this will not deter some less prepared students from enrolling:* $\frac{\partial \overline{q}_1}{\partial w} < 0$.

<u>Note</u>. The absolute rise in private college enrollment $\frac{\partial \overline{q}_1}{\partial w} < 0$ allows for the possibility that the private college enrollment does shrink as the share of the overall college population, i.e., $\left|\frac{\partial \overline{q}_1}{\partial w}\right| < \left|\frac{\partial \overline{q}_2}{\partial w}\right|$, consistent with the facts presented in Table 1.

The Intuition

Increased college premium makes college education attractive for an additional segment of less prepared students for whom this was not the case before. College 2, the “people’s university”, which has a priority to reach more students, is therefore compelled to adjust its curricular standard downward.

This will come at the expense of human capital attainment of the top ability students bound for College 2. For a subset of these students, this will shift the trade-off between the colleges toward college 1. Thus the competition faced by College 1 will become somewhat weaker, so it will be able to afford giving less attention to human capital gains by its lower marginal cohort. Instead, College 1 will raise curricular standard to the benefit of its better students.

The results concerning student outcomes can be summarized as follows.

Corollary. *Under the provisions of the Theorem, the rise of college premium will have a positive effect on the learning outcomes for students in the lower segment of the ability distribution of college 2 as well as those in the upper segment of the ability distribution of college 1 students. The change will have a negative effect on students in the middle, i.e., between the two aforementioned segments.*

Conclusions

We have developed a model of non-linear human capital gains in college, which derives from the extent of the fit between a student's preparation and the college's discretionary span of its curricular plan.

We apply the model to analyze the competition between more selective colleges, which value exclusively the quality of their student population, and the less selective ones that balance quality vs. the quantity of their students. The competition amounts to the choice of the optimal location by the colleges along the axis of student ability (preparation).

We show that as a result of this intercollegiate competition, the rise in college premium will cause less selective colleges to lower their standards further, while the effect on more selective colleges is the opposite: they further increase their selectivity.

The resulting distribution of human capital attainment will feature gains by students at the opposite ends of the ability distribution with stagnation and even relative decline in the middle. Thus an additional insight that can be derived from our analysis is that while the elite colleges become even more selective, they do not necessarily improve the match between their curricular standards and students' preparation in response to rising college premium. This implies that the adjustments triggered by the increased competition between the colleges may in fact lead to inferior human capital attainment by a significant "middle" segment of student population, consistent with the findings by Bound et al. (2009).

A potential extension of our analysis could be obtained from a model incorporating student effort as a factor in human capital attainment. We conjecture that when the overall fit between students and curricula is lowered, this will cause a secular decline in students' effort, consistent with findings in the recent empirical literature such as Babcock and Marks (2011).

Appendix

Proofs (to be completed)

We impose the following condition on the parameters of human capital production and skill premium. Specifically, we posit that the following parametric restriction holds.

Assumption 3. The college premium of the student who gains the highest value added from college 2 curriculum will be no less than 100% relative to (i.e., his earning will be at least double those of) the individual on the margin of attending or not attending college. According to relationships (1) and (2) and Assumption 2, this can be expressed as the following inequality:

$$\frac{b+1}{2}\bar{h}_2 + Aw\frac{(b-1)^2}{4}\bar{h}_2^{\,2} \geq (b+1)\bar{h}_2 + 2\,, \quad \text{or} \quad Aw\frac{(b-1)^2}{4}\bar{h}_2^{\,2} \geq \frac{b+1}{2}\bar{h}_2 + 2$$

<u>Optimization by college 1</u>

In light of the assumption of the uniform distribution of student preparation q , as well as due to formula (6), one can rewrite the optimization problem (4) solved by college 1 as follows:

$$\max_{\bar{h}_1} F_1(\bar{h}_1) = \int_{\frac{b}{b+1}(\bar{h}_1+\bar{h}_2)}^{1} [-q^2 + q(1+b)\bar{h}_1 - b\bar{h}_1^{\,2}]\, dq \tag{A.1}$$

Differentiating this integral as a function of $\bar{h}_1$ (both in the integrand and in the lower limit of integration) we obtain the first order condition of optimum as:

$$F_1'(\bar{h}_1) = -\frac{b}{b+1}\left[-\frac{b^2}{(b+1)^2}(\bar{h}_1+\bar{h}_2)^2 + b\bar{h}_1(\bar{h}_1+\bar{h}_2) - b\bar{h}_1^{\,2}\right] + \int_{\frac{b}{b+1}(\bar{h}_1+\bar{h}_2)}^{1} [q(1+b) - 2b\bar{h}_1\,]\, dq =$$

$$= \frac{b^3}{(b+1)^3}(\bar{h}_1+\bar{h}_2)^2 - \frac{b^2}{b+1}\bar{h}_1\bar{h}_2 + \frac{b+1}{2} - \frac{b^2}{2(b+1)}(\bar{h}_1+\bar{h}_2)^2 - 2b\bar{h}_1 + \frac{2b^2}{b+1}\bar{h}_1(\bar{h}_1+\bar{h}_2)$$

Straightforward algebraic transformations then yield:

$$F_1'(\overline{h}_1) = \frac{b^2}{2(b+1)^3}\left[\overline{h}_1^{\,2}(3b^2+8b+3)+4b\overline{h}_1\overline{h}_2-(b^2+1)\overline{h}_2^{\,2}\right]+\frac{b+1}{2}-2b\overline{h}_1 \qquad \text{(A.2)}$$

hence the first order necessary condition of optimum:

$$\overline{h}_1^{\,2}b^2(3b^2+8b+3)+4b^3\overline{h}_1\overline{h}_2-4b(b+1)^3\overline{h}_1 - b^2(b^2+1)\overline{h}_2^{\,2}+(b+1)^4=0 \qquad \text{(A.3)}$$

Differentiating the expression (A.2) we get

$$F_1''(\overline{h}_1) = \frac{b^2}{(b+1)^3}\left[\overline{h}_1(3b^2+8b+3)+2b\overline{h}_2\right]-2b \qquad \text{(A.4)}$$

and then applying inequality (9) along with the obvious condition $\overline{h}_1 \geq \overline{h}_2$ we obtain the fact

Lemma 3. $F_1''(\overline{h}_1) < 0$ is true at optimum for any b > 1.

This means that equality (A.3) is both necessary and sufficient for optimum in problem (4) (which automatically exists on the compact set of feasible solutions).

Differentiating (A.4) implicitly with respect to $\overline{h}_2$ we obtain

$$\frac{\partial \overline{h}_1}{\partial \overline{h}_2} = \frac{b^2}{(b+1)^3}\left[(b^2+1)\overline{h}_2-2b\overline{h}_1\right]\left(F_1''(\overline{h}_1)\right)^{-1} \qquad \text{(A5)}$$

The sign of this expression characterizes the slope of college 1 reaction curve to a *change* in the curriculum choice of college 2. To analyze it, consider student ability level $\hat{q}_2 = (b+1)\overline{h}_2/2$ at which, as defined earlier, college 2 curriculum yields the highest value added (the ability level at which parabola $f(q,\overline{h}_2)$, defined similarly to (8), reaches its maximum). There are the following two cases (loci of the reaction curve) which depend on the proximity of prior curricular choices of the two colleges 1 and 2.

Locus A. This student (of ability $\hat{q}_2$) will actually attend college 2 (which means that there is indeed non-monotonicity in student gains from the college, in terms of their ability). This implies that $\hat{q}_2 < \overline{q}_1$, or equivalently, according to (6), $\frac{b+1}{2}\overline{h}_2 < \frac{b}{b+1}(\overline{h}_1 + \overline{h}_2)$, i.e.,

$$(b^2+1)\overline{h}_2 < 2b\overline{h}_1 \tag{A6}$$

which means that the gap between the curricular standards of the two colleges is relatively wide. Combined with Lemma 3, inequality (A6) implies that

$$\frac{\partial \overline{h}_1}{\partial \overline{h}_2} > 0 \tag{A7}$$

Locus B. The opposite is true, i.e., the maximum of parabola $f(q,\overline{h}_2)$ is reached to the right of $\overline{q}_1$, or $\hat{q}_2 > \overline{q}_1$. According to (5), this means that $\frac{b+1}{2}\overline{h}_2 > \frac{b}{b+1}(\overline{h}_1 + \overline{h}_2)$, i.e.,

$$(b^2+1)\overline{h}_2 > 2b\overline{h}_1 \tag{A8}$$

so the gap between the curricular standards of the two colleges is relatively small. Combined with Lemma 3, inequality (A8) implies that

$$\frac{\partial \overline{h}_1}{\partial \overline{h}_2} < 0 \tag{A9}$$

The synthesis of the two cases implies that when the curriculum choice $\overline{h}_2$ of college 2 rises and moves closer to $\overline{h}_1$, the slope of college 1 reaction curve turns from positive to negative, i.e., it has the inverted U shape overall.

To obtain the results stated in the Theorem, the equilibrium must occur in the downward sloping portion of college 1 reaction curve. As we will see shortly, the reaction curve of college 2 is positively sloped, which ensures single crossing of the two reaction curves.

Optimization by college 2

Similarly to handling the optimization problem of college 1 while using formula (6) we rewrite the problem (10) solved by college 2 as follows:

$$\max_{\bar{h}_2} F_2(\bar{h}_2) = \int_{\bar{q}_2}^{\frac{b}{b+1}(\bar{h}_1+\bar{h}_2)} [-q^2 + q(1+b)\bar{h}_2 - b\bar{h}_2^2]\, dq + \gamma\left[\frac{b}{b+1}(\bar{h}_1+\bar{h}_2) - \bar{q}_2\right] \qquad \text{(A.10)}$$

where $\bar{q}_2$ is defined by formula (12).

Differentiating the above objective function with respect to $\bar{h}_2$ and using equality (11) we obtain the first order condition of optimum as:

$$\begin{aligned}
0 = F_2'(\bar{h}_2) &= \frac{b}{b+1}\left[-\frac{b^2}{(b+1)^2}(\bar{h}_1+\bar{h}_2)^2 + b\bar{h}_2(\bar{h}_1+\bar{h}_2) - b\bar{h}_2^2\right] - \frac{\partial \bar{q}_2}{\partial \bar{h}_2}\left[-\bar{q}_2^{\,2} + (b+1)\bar{h}_2\bar{q}_2 - b\bar{h}_2^2\right] \\
&+ \int_{\bar{q}_2}^{\frac{b}{b+1}(\bar{h}_1+\bar{h}_2)} [q(b+1) - 2b\bar{h}_2]\, dq + \gamma\left(\frac{b}{b+1} - \frac{\partial \bar{q}_2}{\partial \bar{h}_2}\right) = \\
&= \frac{b^2(b^2+1)}{2(b+1)^3}(\bar{h}_1+\bar{h}_2)^2 - \frac{b^2}{b+1}\bar{h}_2^{\,2} - \frac{b^2}{b+1}\bar{h}_2(\bar{h}_1+\bar{h}_2) - \frac{b+1}{2}\bar{q}_2^{\,2} + 2b\bar{h}_2\,\bar{q}_2 + \frac{\gamma b}{b+1} + \qquad \text{(A11)} \\
&+ \frac{\partial \bar{q}_2}{\partial \bar{h}_2}\left[\bar{q}_2^{\,2} - (b+1)\bar{h}_2\bar{q}_2 + b\bar{h}_2^2 - \gamma\right] \\
&= \frac{b^2}{2(b+1)^3}\left[(b^2+1)\bar{h}_1^{\,2} - (3b^2+8b+3)\bar{h}_2^{\,2} - 4b\bar{h}_1\bar{h}_2\right] - \frac{b+1}{2}\left[(b+1)\bar{h}_2\bar{q}_2 - b\bar{h}_2^{\,2} - (Aw)^{-1}\right] + \\
&+ 2b\bar{h}_2\,\bar{q}_2 + \frac{\gamma b}{b+1} - \frac{\partial \bar{q}_2}{\partial \bar{h}_2}\left[(Aw)^{-1} + \gamma\right]
\end{aligned}$$

We now differentiate (A11) to obtain the second derivative:

$$F_2''(\overline{h}_2) = -\frac{b^2}{(b+1)^3}(3b^2+8b+3)\overline{h}_2 - \frac{2b^3}{(b+1)^3}\overline{h}_1 + b(b+1)\overline{h}_2 - \frac{(b-1)^2}{2}\overline{q}_2 - \frac{(b-1)^2}{2}\overline{h}_2\frac{\partial\overline{q}_2}{\partial\overline{h}_2} - \\ -\frac{\partial^2\overline{q}_2}{\partial\overline{h}_2^{\,2}}\left[(Aw)^{-1}+\gamma\right] \tag{A12}$$

Differentiating expression (12) we obtain

$$\frac{\partial\overline{q}_2}{\partial\overline{h}_2} = 2^{-1}\left[b+1-\frac{(b-1)^2\overline{h}_2}{\sqrt{(b-1)^2\overline{h}_2^{\,2}-4(Aw)^{-1}}}\right] \tag{A13}$$

$$\frac{\partial^2\overline{q}_2}{\partial\overline{h}_2^{\,2}} = \frac{2(b-1)^2(Aw)^{-1}}{\left[(b-1)^2\overline{h}_2^{\,2}-4(Aw)^{-1}\right]^{3/2}} > 0 \tag{A14}$$

Now observe that the inequality in Assumption 3 implies that

$$4(Aw)^{-1} < \frac{(b-1)^2}{2}\overline{h}_2^{\,2} \tag{A15}$$

Applying (A15) to (A13) and (12) we can write, respectively

$$\frac{\partial\overline{q}_2}{\partial\overline{h}_2} > 2^{-1}\left[b+1-\sqrt{2}(b-1)\right] \tag{A16}$$

$$\overline{q}_2 < 2^{-1}\left[b+1-(b-1)/\sqrt{2}\right]\overline{h}_2 = 4^{-1}\left[2(b+1)-\sqrt{2}(b-1)\right]\overline{h}_2 \tag{A17}$$

We now apply inequalities (A17), (A16) and (A14) as well as the fact $\overline{h}_2 < \overline{h}_1$ to the formula (A12) and obtain the following estimate valid for any positive value of γ.

$$F_2''(\overline{h}_2) < -\frac{b^2}{(b+1)^3}(3b^2+10b+3)\overline{h}_2 + b(b+1)\overline{h}_2 - \frac{(b-1)^2}{2}\overline{h}_2 + \\ -\frac{(b-1)^2}{4}\left[b+1-\sqrt{2}(b-1)\right]\overline{h}_2 = \frac{P(b)}{(b+1)^3}\overline{h}_2$$

where *P(b)* is polynomial of parameter b whose direct examination shows that it is negative for all values of $b \geq 3$.

We now differentiate equation (A11) implicitly with respect to $\bar{h}_1$, keeping in mind that according to (12) $\bar{q}_2$ does not directly depend on $\bar{h}_1$, and obtain the relationship

$$\frac{\partial \bar{h}_2}{\partial \bar{h}_1} = \frac{b^2}{(b+1)^3}\left[2b\bar{h}_2 - (b^2+1)\bar{h}_1\right]\left(F_2''(\bar{h}_1)\right)^{-1} \tag{A18}$$

It is clear that the term $2b\bar{h}_2 - (b^2+1)\bar{h}_1$ is negative since $\bar{h}_2 \leq \bar{h}_1$. Combined with the established negativity of $F_2''(\bar{h}_2)$ this yields the fact

$$\frac{\partial \bar{h}_2}{\partial \bar{h}_1} > 0 \tag{A19}$$

Implying that the reaction curve of college 2 is positively sloped.

References

Autor, David, Lawrence Katz, and Melissa Kearney (2008). “Trends in U.S. Wage Inequality: Re-Assessing the Revisionists,” *Review of Economics and Statistics* 90, 300–323.

Babcock, Philip and Mindy Marks (2011). “The Falling Time Cost of College: Evidence from Half a Century of Time Use Data,” *Review of Economics and Statistics* 93, 468-478.

Betts, Julian (1998). “The Impact of Educational Standards on the Level and Distribution of Earnings,” *American Economic Review* 88, 266–275.

Blankenau, William (2005). “Public Schooling, College Subsidies and Growth,” *Journal of Economic Dynamics and Control* 29, 487–507.

Bound, John, Brad Hershbein and Bridget Terry Long (2009). "Playing the Admissions Game: Student Reactions to Increasing College Competition," *Journal of Economic Perspectives* 23, 119-146.

Costrell, Robert (1994). “A Simple Model of Educational Standards,” *American Economic Review* 84, 956–971.

Digest of Education Statistics.

Epple, Dennis, Elizabeth Newlon, and Richard Romano (2002). “Ability Tracking, School Competition, and the Distribution of Educational Benefits,” *Journal of Public Economics* 83, 1–48.

Epple, Dennis, Richard Romano, and Holger Sieg (2006). “Admission, Tuition, and Financial Aid Policies in the Market for Higher Education,” *Econometrica* 74, 885–928.

Gilpin, Gregory and Michael Kaganovich (2012). “The Quantity and Quality of Teachers: Dynamics of the Trade-off,” *Journal of Public Economics* 96, 417-429.

Hoxby, C. M. (2009) “The Changing Selectivity of American Colleges,” *Journal of Economic Perspectives* 23(4), 95-118.

Hoxby, C. M. (2012) “Endowment Management Based on a Positive Model of the University, ” NBER, Working Paper 18626.

Kaganovich, Michael and Xuejuan Su (2013). “College Expasion and Curriculum Choice”, mimeo.

Su, Xuejuan (2004). “The Allocation of Public Funds in a Hierarchical Educational System,” *Journal of Economic Dynamics and Control* 28, 2485–2510.

Rothschild, M. and L.J. White (1995) “The Analytics of Pricing in Higher Education and Other Services in Which Customers are Inputs,” *Journal of Political Economy* 103, 573-86.

van Geert, P. (1998). “A Dynamic Systems Model of Basic Developmental Mechanisms: Piaget, Vygotsky, and beyond,” *Psychological Review* 105, 634-677.

van Geert, P. and H. Steenbeek (2005). “The Dynamics of Scaffolding,” *New Ideas in Psychology* 23, 115-128.

Winston, G.C. (1999) “Subsidies, Hierarchy and Peers: The Awkward Economics of Higher Education,” *Journal of Economic Perspectives* 13(1), 13-36.