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Alliances and strategic advantage in sequential-move contests: Implications for offensive vs. defensive strategies

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ABSTRACT

This paper examines the impact of alliances and moving order on strategic advantage, conflict intensity, and expected payoffs in three-player sequential-move contests. The study shows that in a scenario where multiple players act as defenders while facing aggression from a lone player that moves first as an attacker, they must make their arming decisions jointly to gain a strategic advantage. Conversely, when attacking a lone player that moves second as a defender, multiple players acting as first movers must make their arming decisions autonomously. Compared to the benchmark equilibrium in a simultaneous-move game, the overall conflict intensity is higher if multiple players arm independently and lower if they arm cooperatively as an alliance. The expected payoffs of all players are the highest in a sequential-move game when two players ally, regardless of their moving order. Based on the analysis, we find that it is effective to launch an *offensive* strategy when allied players make arming decisions autonomously and strike first as attackers. On the other hand, a *defensive* strategy is effective when allied players make arming decisions collectively and move second as defenders.

1. Introduction

Issues regarding the effectiveness of forming a strategic alliance and the situations in which players have an advantage over opponents have been significant concerns in market competition and non-market rivalry.[1] In non-market rivalry, among the critical questions that call for further research are the following: When defending against an attack by a lone player, under what circumstances would multiple players' *defensive* strategies effectively enhance their chance of success? In a different situation, what are the conditions under which multiple players' *offensive* tactics against a lone player can be successful? Are an alliance's defensive and offensive strategies different or similar? The objectives of this paper are to investigate these questions from a conflict-theoretic perspective.

Researchers point out that in a two-player contest, one of the players might discover it advantageous to adopt the first-mover or the first-strike strategy (e.g., Dixit 1987; Baik and Shogren 1992; Leininger 1993; Linster 1993; Gershenson and Grossman 2000; Morgan 2003; Aanesen 2011). A player who makes the first move or decides to wait and observe their opponent's action can fundamentally

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[1] See, e.g., Bagwell and Wolinsky (2002) and Corchón and Marini (2018).

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alter the game's equilibrium outcome. This indicates that the timing of a player's move is critical when determining the optimal effort in a sequential-move game of the Stackelberg type, which differs from the effort dictated by the Nash equilibrium (Congleton et al., 2008).

Several studies show that the moving order may strategically alter the equilibrium outcomes of a two-player contest with complete information. For example, Morgan (2003) evaluates the expected payoffs of two players in sequential-move games and simultaneous-move games. The author shows that sequential-move games may be preferred as the expected payoffs of the players are higher. According to Jost and Krakel's (2005) analysis, players' strategies in sequential-move tournaments are quite different from simultaneous-move tournaments. The first mover in a sequential-move tournament makes a preemptively strong effort to force the second mover to give up. Chang et al. (2018) use the framework of three-player sequential-move games to show that the intervention of a third party may strategically alter the equilibrium outcomes of a two-player conflict.[2] Nelson (2020) investigates whether equilibrium results in two-player and three-player sequential contests differ from one another in a lab experiment setting. The study discovers that while the Stackelberg leader in two-player sequential contests has no strategic advantage, in sequential contests with multiple leaders, this is no longer the case.

In this paper, we examine how forming an alliance and allied players' moving order jointly affect the equilibrium outcomes of arming allocations, conflict intensity, and strategic advantage in three-player sequential-move contests. The main findings are as follows: Firstly, multiple players acting as first movers (i.e., attackers) have a strategic advantage over a stand-alone follower (i.e., defender) only when the attackers make their arming decisions *independently*. If, instead, the multiple attackers form an alliance and make their arming decisions cooperatively, they no longer have the strategic advantage. The formation of an alliance requires that member players make their decisions cooperatively. Such decisions reduce each player's arming (or effort), causing the loss of their first-strike advantage.

Secondly, a stand-alone player that moves first as attacker has a strategic advantage when multiple players that move second as defenders make their arming decisions non-cooperatively or independently. Thirdly, compared to a simultaneous-move game with three independent players, the overall conflict intensity is higher (lower) when the multiple players arm independently (cooperatively as an alliance). The three players' expected payoffs are the highest when the multiple players form an alliance, irrespective of whether they are first or second movers.

Research employing sequential-move games to analyze various issues in contests and conflicts is on the rise. For instance, Shinkai (2000) investigates a three-player Stackelberg game with private information and discovers that players' payoffs can be non-monotonic. His analysis highlights the challenges in characterizing first-mover or later-mover advantages in sequential-move oligopolistic competition. Konrad and Leininger (2007) examine two-stage, multi-player all-pay auctions with complete information. They find that a player's expected payoff is influenced by their effort costs and their position in relation to other competitors. Utilizing an experimental approach, Jian et al. (2017) model contests as incomplete information all-pay auctions with linear costs. Their findings indicate that expected effort is higher in simultaneous-move contests compared to sequential-move contests.

Hausken and Bier (2011) examine the interactions that emerge when a single defender confronts multiple independent attackers. In particular, the authors investigate the circumstances in which a lone defender thwarts the danger of multiple independent attackers in three moving-order situations. In the first scenario, the defender and the attackers fight in a simultaneous-move game. In the second scenario, the defender moves first, and the multiple attackers move second. In the third scenario, the multiple attackers move first, and the defender moves second. They discover that (i) every agent continues to actively participate in a simultaneous-move game. (ii) In a sequential-move game, the defender deters all attackers when it moves first in a sequential game. (iii) When multiple heterogeneous attackers move first, the stronger attacker drives other attackers to withdraw from the fight since the most powerful attacker will be able to seize significant advantage of the defender's asset.

Our paper builds upon the work of Hausken and Bier (2011) in three key areas. First, we explore the effects of multiple players forming an alliance to attack a single defender. We focus on differences in outcomes, such as the allocation of resources for armed conflict between the attackers and the lone defender, their expected payoffs, and the overall intensity of the conflict. Second, we analyze the resource allocations that occur when a stand-alone player takes the initiative to attack multiple players. These players must decide whether to arm themselves independently or to form a military alliance for joint arming decisions. Third, we investigate the conditions under which offensive strategies (first mover advantage) and defensive strategies (second mover position) are effective. This analysis occurs in the context of multiple attackers and defenders when they choose to arm either independently or as part of an alliance.

The situation surrounding Russia's invasion of Ukraine can be compared to a sequential-move game. Russia recognized that there was no military alliance between Ukraine and NATO (or the United States), leading them to believe they could take the initiative and start a war against Ukraine. This belief aligns with the concept of having a strategic advantage in a sequential-move game. Our analysis indicates that the attacker perceives a first-mover advantage when multiple potential second-movers do not cooperate in making defensive armament decisions. After Russia invaded Ukraine, the country started receiving military weapons from Western nations, primarily led by the United States and NATO. In scenarios where Ukraine and its military supporters could make their own decisions about arming themselves as aggressors, Russia shifted to a defensive position and consequently lost its strategic advantage.

[2] For studies that examine the effect of strategic third-party intervention on a two-party conflict see, e.g., Blechman (1995), Regan (1996, 2002), Siqueira (2003), Rowlands and Carment (2006), Chang et al. (2007), Chang and Sanders (2009), and Chang and Sellak (2022).

Unexpectedly, the conflict between Russia and Ukraine has grown into a prolonged war.[3]

The present study complements the contribution of Ke et al. (2013), which examines a contest between a two-player alliance and a standalone enemy. Here are some key differences between the two analyses. Ke et al. (2013) explore how the possibility of "future re-distributional conflict" influences the allocation of arms between the two allied players during their fight against a common enemy. The researchers demonstrate that after a lone player is defeated, "the experience of fighting's shoulder-to-shoulder' in an alliance" against the common enemy does not necessarily reduce their conflict over how to split the rewards. In our research, we focus on the distinctive implications between offensive and defensive strategies within a sequential-move game involving the two-player alliance and their common enemy. Our findings indicate that an offensive strategy is effective when allied players arm themselves independently and take the initiative as attackers. Conversely, a defensive strategy proves effective when they make their arming decisions collectively and act as defenders after the enemy has made its move.

It is instructive to highlight some interesting studies on offensive and defensive strategies and their effects on conflict. Evera (1998) analyzes the relationship between the offense and defense balance in military capabilities and the likelihood of war. By examining historical case studies from Europe, Ancient China, and the United States, the author identifies factors that intensify war risk when offense dominates. Evera (1998) also demonstrates that military-defensive alliances can help mitigate these risks and reduce the likelihood of conflict. Gortzak et al. (2005) conduct an empirical study on how shifts in offensive and defensive military strategies impact the likelihood of war and militarized interstate disputes (MIDs). Using time series data from MIDs in the international system between 1816 and 1992, the three researchers find that changes in the balance of offensive and defensive strategies may have limited utility as a predictor of war and militarized disputes on a systemic level. However, these shifts could still be relevant at the dyadic or regional level. Gortzak et al. (2005) conclude that other factors, such as power distribution and regime type, are stronger predictors of conflict.

Robert (2014) examines empirically how third-party military support would affect shifts between defensive and offensive military strategies in civil wars. The findings reveal that offensive military support from external powers tends to reduce the duration of peace, while defensive military support tends to prolong it. Beard and Strayhom (2018) analyze possible factors that contribute to the first-strike advantage, leading to prolonged and destructive wars. The researchers decompose the first-strike advantage into three key components: offensive tactical advantage, mobilization advantage, and the destructive effects of an initial attack. By analyzing data from the 1967 and 1973 Arab-Israeli wars, Beard and Strayhom (2018) present several significant findings. Offensive tactical advantages can trigger war when initial attacks severely degrade enemy capabilities. Mobilization advantages consistently contribute to achieving a first-strike capability, and when combined with defensive advantages, they are likely to increase the probability of prolonged wars.

The remainder of the paper is organized as follows: Section 2 lays out the benchmark model of a three-player simultaneous-move contest. Section 3 examines sequential-move contests between two attackers or defenders and a common enemy in the absence of alliance formation. Section 4 analyzes situations where two attackers or defenders form an alliance against a stand-alone player. Overall comparisons between alliance and non-alliance are shown in Section 5. Section 6 concludes.

2. The analytical framework and the benchmark equilibrium

We first look at the benchmark case in which three identical, risk-neutral players (denoted by A, B, and C) compete for an indivisible prize $V(>0)$ and make their arming decisions independently in a simultaneous-move contest. Each player allocates $G_i(>0)$amount of resource to arming, where $i = A, B, C$. The probability that player i wins the contest is given by the contest success function (CSF) *à la* Tullock (1980):

$$p_i = \frac{G_i}{G_A + G_B + G_C} \text{ for } i = A, B, C.$$

The objective of each player is to determine an arming allocation that maximizes its expected payoff. That is, player i solves the following optimization problem:

$$\underset{\{G_i\}}{Max}\pi_i = \frac{G_i}{G_A + G_B + G_C} V - G_i.$$

The first-order condition (FOC) implies that each player's arming allocation is:

$$G_A^* = G_B^* = G_C^* = \frac{2V}{9}. \quad (1)$$

Conflict intensity is taken as the sum of the players' arming allocations:

$$CI^* = G_A^* + G_B^* + G_C^* = \frac{2V}{3}. \quad (2)$$

[3] For future study, a more complicated model is necessary to systematically explain the duration of the Russia-Ukraine war and other related issues.

In equilibrium, each player's expected payoff is calculated as[4]

$$\pi_i^* = \frac{G_i^*}{G_A^* + G_B^* + G_C^*} V - G_i^* = \pi\left(\frac{2V}{9} \Big/ \frac{2V}{3}\right) V - \frac{2V}{9} = \frac{V}{9}. \tag{3}$$

The equilibrium results of the simultaneous-move contest, as shown in (1)-(3), serve as the benchmark to evaluate outcomes under different scenarios.

The different scenarios we analyze involve three players engaging in sequential-move contests and two of the players may or may not ally. For the purpose of our analyses, we define a player that moves or strikes first as an "attacker" in that it is an aggressor in starting a fight or war. In contrast, a player that moves second is a "defender" in fighting back against aggression. We consider that two players (say, A and B) jointly decide on their timing positions as either first or second movers.

As for determining the amount of resources allocated to fighting, two players that form an alliance may choose to make their decisions autonomously or collectively. This approach allows us to separate allied players' arming decisions (autonomous versus collective) from their moving order (jointly moving first as attackers or second as defenders) to be two different elements of decision-making in sequential contests.

3. (No alliance) multiple arming-independent players as attackers or defenders in sequential-move contests

In this section, we look at two sequential-move contests. In the first scenario (see Section 3.1), two players decide to act as first movers (i.e., Stackelberg leaders) in launching an attack while the lone player acts as a second mover (i.e., a Stackelberg follower) in defending. In the second scenario (see Section 3.2), the lone player acts as the first mover in attacking, while two other players act as second movers in defending. We do not take into account the formation of an alliance between the first movers or the second moves when analyzing and comparing the two different contests. That is, the multiple attackers or defenders are not allies and are free to determine their arming allocations independently.[5] Our aim is to use the benchmark equilibrium in the simultaneous-move contest to evaluate these two sequential-move contests (see Section 3.3).

3.1. Arming-independent players move first in attacking a stand-alone player moving second

We use the superscript "MLN" to represent the case of multiple leaders (ML) as attackers, A and B, and a single defender, C, in a sequential-move game with no (N) alliance. As standard in game theory, we use backward induction to derive the subgame-perfect Nash equilibrium.

We show in Appendix A-2 that the equilibrium arming allocations of the multiple attackers and the sole defenders are given, respectively, as follows:

$$G_A^{MLN} = G_B^{MLN} = \frac{9V}{32} \text{ and } G_C^{MLN} = \frac{3V}{16}. \tag{4}$$

It follows from (4) that the differences in arming allocations are:

$$G_C^{MLN} - G_j^{MLN} = \frac{3V}{16} - \frac{9V}{32} = -\frac{3V}{32} < 0 \text{ for } j = A, B;$$

$$G_C^{MLN} - \left(G_A^{MLN} + G_B^{MLN}\right) = \frac{3V}{16} - \left(\frac{9V}{32} + \frac{9V}{32}\right) = -\frac{3V}{8} < 0.$$

That is,

$$G_A^{MLN} > G_C^{MLN} \text{ or } \left(G_A^{MLN} + G_B^{MLN}\right) > G_C^{MLN}.$$

The arming allocation of each attacker is strictly greater than that of the single defender, noting that the two attackers make their arming decisions independently. This implies that:

$$p_j^{MLN} > p_C^{MLN} \text{ for } j = A, B.$$

These results indicate that players A and B move first as attackers have a strategic advantage when each attacker has the discretion to determine its optimal arming level.

In equilibrium, the overall conflict intensity is:

$$CI^{MLN} = G_A^{MLN} + G_B^{MLN} + G_C^{MLN} = \frac{9V}{32} + \frac{9V}{32} + \frac{3V}{16} = \frac{3V}{4}, \tag{5}$$

and the expected payoffs of the three players are:

[4] See Appendix A-1 for detailed calculations for the equilibrium results.

[5] The case of alliance formation will be addressed in Section 4.

$$\pi_A^{MLN} = \pi_B^{MLN} = \frac{3V}{32} \text{ and } \pi_C^{MLN} = \frac{V}{16}. \tag{6}$$

It follows from (6) that:

$$\pi_C^{MLN} - \pi_j^{MLN} = \frac{V}{16} - \frac{3V}{32} = -\frac{V}{32} < 0 \text{ for } j = A, B.$$

It comes as no surprise that each attacker's expected payoff exceeds that of a defender.

The next step is to compare the equilibrium outcomes between the simultaneous-move game (see Section 2) and the sequential-move game when multiple players move first as attackers and make their arming allocations autonomously. Following from (1) and (4), the differences in the arming allocations of the players are:

$$G_j^* - G_j^{MLN} = \frac{2V}{9} - \frac{9V}{32} = -\frac{17V}{288} < 0 \text{ for } j = A, B, \tag{7a}$$

$$G_C^* - G_C^{MLN} = \frac{2V}{9} - \frac{3V}{16} = \frac{5V}{144} > 0. \tag{7b}$$

As for the difference in the overall intensity of conflict, we have from (2) and (5) that:

$$CI^* - CI^{MLN} = \frac{2V}{3} - \frac{3V}{4} = -\frac{V}{12} < 0 \tag{8}$$

According to the results in (3) and (6), the differences in the expected payoffs are:

$$\pi_j^* - \pi_j^{MLN} = \frac{V}{9} - \frac{3V}{32} = \frac{5V}{288} > 0 \text{ for } j = A, B; \tag{9a}$$

$$\pi_C^* - \pi_C^{MLN} = \frac{V}{9} - \frac{V}{16} = \frac{7V}{144} > 0. \tag{9b}$$

Based on the results in (7)-(9), we summarize their economic implications in the first proposition:

PROPOSITION 1. Compared to the simultaneous-move contest, a sequential-move contest in which two arming-independent players move first as attackers and a lone defender moves second has the following outcomes: (i) the overall conflict intensity is higher; (ii) each player's expected payoff is lower; and (iii) the arming allocation of each first-mover is <u>higher,</u> whereas that of the second-mover as defender is <u>lower</u>. As a result, the arming-independent attackers have a strategic advantage over the stand-alone defender, even though the multiple players arm independently.

Proposition 1 indicates that multiple attackers that make their arming decisions autonomously have a strategic advantage in fighting against a single player as a defender. This result stands in contrast with the finding in a sequential-move contest with two players, one attacker and one defender, where a single attacker does not have a strategic advantage in fighting against a single defender. The results in Proposition 1 are consistent with the findings of Nelson (2020) in an experimental study. We extend the analysis to the case of a single attacker and multiple defenders and the presence of an alliance at the leadership and followership stages of the sequential-move game.

The findings of Proposition 1 have the economic implications as follows. The increased arming allocations of multiple players acting independently in aggression and the decreased arming allocation of the lone player in defense can be attributed to the attacker's greater motivation to maintain dominance over the defender and each other in case a future victory of one attacker turns over another attacker. Consequently, this leaves the lone player that moves second as a defender with a strategic disadvantage, as can be seen in the Gulf War, where Iraq, as a stand-alone state, faced a coalition of forces. The coalition members autonomously or independently increased their military expenditures, while Iraq struggled to match that increase with little or no chance of victory.[6]

3.2. A lone player moves first in attacking multiple arming-independent defenders

We proceed to analyze the effects on arming allocations, conflict intensity, and expected payoff when a single player is a first mover in attacking two other players that move second as defenders and make their arming decisions non-cooperatively. We use the superscript "*SLN*" to denote the case of a stand-alone leader (*SL*) as an attacker and multiple players as defenders with no (*N*) alliance in a sequential-move game. The structure of the sequential-move contest is as follows: at stage one, player C makes its arming decision as an attacker. At stage two, players A and B move second and make their arming decisions independently.

Using backward induction, we show in Appendix A-3 that the equilibrium arming allocations of the three players are:

$$G_A^{SLN} = G_B^{SLN} = \frac{3V}{16} \text{ and } G_C^{SLN} = \frac{3V}{8}. \tag{10}$$

It follows from (10) that:

[6] See, e.g., the study by Eichenberg (2005).

$$G_C^{SLN} - G_j^{SLN} = \frac{3V}{8} - \frac{3V}{16} = \frac{3V}{16} > 0 \text{ for } j = A, B.$$

The above inequality indicates that player C has a first-mover strategic advantage when players A and B arm independently and move second as defenders.

In equilibrium, the overall intensity of conflict is:

$$CI^{SLN} = G_A^{SLN} + G_B^{SLN} + G_C^{SLN} = \frac{3V}{4}, \tag{11}$$

and the expected payoffs of the three players are:

$$\pi_A^{SLN} = \pi_B^{SLN} = \frac{V}{16} \text{ and } \pi_C^{SLN} = \frac{V}{8}. \tag{12}$$

Next, we compare the equilibrium outcomes between the simultaneous-move contest and the sequential-move contest, in which a single player moves first in attacking multiple arming-independent players as defenders. We have from the results in (1) and (10) that the differences in the arming allocations are:

$$G_j^* - G_j^{SLN} = \frac{2V}{9} - \frac{3V}{16} = \frac{5V}{144} > 0 \text{ for } j = A, B, \tag{13a}$$

$$G_C^* - G_C^{SLN} = \frac{2V}{9} - \frac{3V}{8} = -\frac{11V}{72} < 0. \tag{13b}$$

As for the difference in the overall conflict intensity, we have from (2) and (11) that:

$$CI^* - CI^{SLN} = \frac{2V}{3} - \frac{3V}{4} = -\frac{V}{12} < 0. \tag{14}$$

According to (3) and (12), the differences in the expected payoffs of the players are:

$$\pi_j^* - \pi_j^{SLN} = \frac{V}{9} - \frac{V}{16} = \frac{V}{144} > 0 \text{ for } j = A, B, \tag{15a}$$

$$\pi_C^* - \pi_C^{SLN} = \frac{V}{9} - \frac{V}{8} = -\frac{V}{72} < 0. \tag{15b}$$

These results in (13)-(15) permit us to summarize their implications in the following proposition:

PROPOSITION 2. Compared to the simultaneous-move game, a sequential-move game in which a single player moves first as an attacker and two arming-independent players move second as defenders has the following outcomes: (i) the overall conflict intensity is higher; (ii) the single player's expected payoff is higher while each of the arming-independent players as defenders has a lower expected payoff; (iii) the single player as an attacker has a higher level of arming allocation while each defender's arming allocation is lower. As a result, the single player attacker, has a strategic advantage to win the game against the arming-independent multiple players.

Proposition 2 indicates that an attacker has a strategic advantage over multiple players that adopt a defensive strategy while making their arming allocations non-cooperatively.

The Russia-Ukraine war may exemplify the principles outlined in Proposition 2. Russia recognized that Ukraine did not have a military alliance with the United States or NATO. As a result, Russia perceived it was advantageous to take the initiative by launching a war against Ukraine. This perception aligns with the concept of strategic advantage in a sequential-move game, as described in Proposition 2.

In a sequential-move game, the player who attacks first may have a significant advantage (i.e., the first-strike advantage). They can exploit the lack of an alliance among the multiple defenders that move second by targeting them individually. This allows the attacker to escalate the conflict more effectively than in a simultaneous-move scenario. A historical example of this scenario can be seen in the Roman Empire's conquests. Rome often faced smaller, independent groups that lacked alliance, making it easier for them to conduct successful military campaigns. Anticipating the challenge of facing multiple defenders, the attacker is likely to invest heavily in military capabilities. In contrast, defenders often find themselves limited by their individual strengths and the absence of coordinated defensive strategies.

The results of Proposition 2 can also be illustrated with another example. Russia's intervention in Syria in 2015 targeted several independent armed groups, including various rebel groups and ISIS. While these groups fought independently against Russia, their lack of alliance presented a strategic advantage for Russia. This disunity allowed Russia to succeed in reaching its strategic objectives in the region.[7]

Proposition 2 emphasizes the strategic advantages a single attacker has against multiple defenders arming autonomously in a

[7] See, e.g., the studies by Pukhov (2017), and Charap et al. (2019).

sequential-move game. The attacker's ability to increase arming and exploit the absence of an alliance between the defenders, which often leads to higher conflict intensity, greater resource allocation for warfare, and a higher expected payoff for the first-mover as an aggressor.

3.3. *Comparing between multiple arming-independent attackers and a lone attacker*

In the absence of arming allies, would multiple players that move first in attacking a stand-alone player necessarily dominate the alternative scenario when the lone player moves first as an attacker? To answer this question, we have from the results in (4) and (10) that the differences in arming allocations are:

$$G_j^{SLN} - G_j^{MLN} = \frac{3V}{16} - \frac{9V}{32} = -\frac{3V}{32} < 0 \text{ for } j = A, B, \tag{16a}$$

$$G_C^{SLN} - G_C^{MLN} = \frac{3V}{8} - \frac{3V}{16} = \frac{3V}{16} > 0. \tag{16b}$$

As for the difference in the overall intensity of conflict, we have from (5) and (11) that:

$$CI^{SLN} - CI^{MLN} = \frac{3V}{4} - \frac{3V}{4} = 0. \tag{17}$$

According to (3) and (12), the differences in the expected payoffs are:

$$\pi_A^{SLN} - \pi_A^{MLN} = \frac{V}{16} - \frac{3V}{32} = -\frac{V}{32} < 0 \text{ for } j = A, B, \tag{18a}$$

$$\pi_C^{SLN} - \pi_C^{MLN} = \frac{V}{8} - \frac{V}{16} = \frac{V}{16} > 0. \tag{18b}$$

The findings in (16)-(18) lead to the implications as summarized in the following proposition:

PROPOSITION 3. In a three-player sequential-move contest, we have the following results: (i) The expected payoff of a player that moves first as an attacker exceeds that of two arming-independent players that move first as attackers; (ii) The overall conflict intensity stays the same, regardless of whether there are multiple attackers or a single attacker; (iii) The first-mover advantage to win a conflict holds, regardless of whether there are multiple attackers or just one attacker

Proposition 3 suggests that, other things being equal, a lone attacker in a three-player conflict has a strategic advantage over two other players adopting a defensive strategy while making their arming decisions independently. The expected payoff for the single attacker is greater, the conflict intensity remains constant, and the first-mover advantage persists, regardless of the number of attackers involved. This proposition highlights the effectiveness of the offensive strategy and suggests the benefit of being the first to attack in a multi-player conflict. In other words, there exists a first-strike advantage.

4. (Alliances) multiple arming-cooperative players as attackers or defenders in sequential-move contests

How would arming allies by a subset of players affect the equilibrium outcomes in three-player sequential-move contests without such an alliance? In this section, we tackle this question. We define alliance formation as a commitment under which allied players make their arming decisions jointly or cooperatively to maximize the sum of their expected payoffs.[8]

We look at two types of contests with arming allies. In the first scenario, two allied players agree to move first as attackers in fighting against a single player that moves second as a defender (see Section 4.1). In the second scenario, a lone player moves first as an attacker, fighting against the two allied players that move second as defenders (see Section 4.2). We then compare the equilibrium outcomes between allied attackers and a single attacker (see Section 4.3).

4.1. *Allied players move first in attacking a lone player that moves second as a defender*

In analyzing a sequential-move contest where two players establish an alliance, we use the superscript "*MLA*" to represent the case of multiple leaders/attackers as allies. At stage one, players A and B move first as attackers and make their arming decisions cooperatively to maximize the joint expected payoff. The CSF of the allied players and their expected payoffs are given, respectively as follows:

[8] Using an experimental approach, Ke et al. (2015) show that coalitions are more likely to establish in the shadow of conflict when players are similar in terms of their willingness to contribute to the joint cause (e.g., fighting against a common enemy) and their respective portions of the prize. For studies on the strategic interaction with alliances or in-group solidarity/hostility see, e.g., Ke et al. (2013). Boudreau et al. (2019) examine a three-party conflict to show the potential impact that noise may have on alliance formation. They discover that two parties may decide to establish an alliance to combine their forces against a rival.

$$P_{AB} = \frac{(G_A + G_B)}{(G_A + G_B) + G_C} \text{ and } \Pi = \pi_A + \pi_B = \frac{(G_A + G_B)}{(G_A + G_B) + G_C} V - G_A - G_B, \tag{19a}$$

where

$$\pi_A = \frac{(G_A + G_B)}{(G_A + G_B) + G_C} \alpha V - G_A \text{ and } \pi_B = \frac{(G_A + G_B)}{(G_A + G_B) + G_C} (1 - \alpha) V - G_B. \tag{19b}$$

Note that α is the share of the prize that player A receives and $1 - \alpha$ is that of the prize that player B receives after winning the conflict. The value of α is taken to be exogenous in order to see its range under which players A and B have incentives to form an alliance for a higher expected payoff.

Beginning the analysis at the second stage of the two-stage game, player C as a defender decides on an optimal arming allocation that maximizes its expected payoff:

$$\pi_C = \frac{G_C}{G_A + G_B + G_C} V - G_C.$$

Plugging player C's arming allocation, $G_C = \sqrt{(G_A + G_B)V} - G_A - G_B$, into the joint expected payoff function of players A and B, $\Pi = \pi_A + \pi_B$ in (19a), we solve for the arming levels of the allied players at the first stage of the game. We show in Appendix A-4 that the subgame Nash-perfect equilibrium levels of arming allocations by the three players are:

$$G_A^{MLA} = \frac{V}{8}, G_B^{MLA} = \frac{V}{8}, \text{and } G_C^{MLA} = \frac{V}{4}. \tag{20a}$$

It follows from (20) that:

$$\left(G_A^{MLA} + G_B^{MLA}\right) - G_C^{MLA} = \left(\frac{V}{8} + \frac{V}{8}\right) - \frac{V}{4} = 0.$$

In equilibrium, the overall conflict intensity is:

$$CI^{MLA} = G_A^{MLA} + G_B^{MLA} + G_C^{MLA} = \frac{V}{8} + \frac{V}{8} + \frac{V}{4} = \frac{V}{2}, \tag{20b}$$

and the expected payoffs of the players are calculated as follows:

$$\pi_A^{MLA} = \frac{V}{8}(4\alpha - 1) > 0 \text{ and } \pi_B^{MLA} = \frac{V}{8}(3 - 4\alpha) > 0 \text{ when } \frac{1}{4} < \alpha < \frac{3}{4}, \tag{21a}$$

$$\pi_C^{MLA} = \frac{V}{4}. \tag{21b}$$

These equilibrium results show that the allied attackers' overall arming allocation is equal to that of the lone defender's, implying that the probability of winning is identical for the ally and its enemy. Accordingly, players A and B do not have the first-mover strategic advantage despite the fact that they form an alliance. Interestingly, the allied attackers have an incentive to form an alliance when the value of α satisfies the following condition: $\frac{1}{4} < \alpha < \frac{3}{4}$.

4.2. A lone player moves first in attacking multiple arming-cooperative defenders

Next, we use the superscript "*SLA*" to denote the case of a stand-alone player moving first as an attacker in fighting against multiple allied players moving second as defenders. At stage one, player C chooses its arming allocation and moves first as an aggressor. At stage two, players A and B move second in defending and make their arming decisions jointly to maximize the sum of their expected payoffs, $\Pi = \pi_A + \pi_B$.

Using backward induction, we show in Appendix A-5 that the subgame Nash-perfect equilibrium levels of arming allocations by the three players are:

$$G_C^{SLA} = \frac{V}{4} \text{ and } G_A^{SLA} = G_B^{SLA} = \frac{V}{8}, \tag{22}$$

which imply that:

$$G_C^{SLA} - \left(G_A^{SLA} + G_B^{SLA}\right) = \frac{V}{4} - \frac{V}{4} = 0.$$

Based on the results in (22), we calculate the equilibrium levels of conflict intensity and expected payoffs as follows:

$$CI_A^{SLA} = G_A^{SLA} + G_B^{SLA} + G_C^{SLA} = \frac{V}{2}, \tag{23}$$

$$\pi_A^{SLA} = \frac{V}{8}(4\alpha - 1) > 0 \text{ and } \pi_B^{SLA} = \frac{V}{8}(3 - 4\alpha) > 0 \text{ when } \frac{1}{4} < \alpha < \frac{3}{4},$$

$$\pi_C^{SLA} = \frac{V}{4}. \tag{24}$$

These results indicate that player C as the attacker does not have the first-mover strategic advantage when players A and B form an alliance and act as second movers in defense. Additionally, the second movers as defenders have an incentive to form an alliance only when the value of α satisfies the following condition:$\frac{1}{4} < \alpha < \frac{3}{4}$.

4.3. Comparing the outcomes between arming-cooperative attackers and a single attacker

We can now compare the equilibrium outcomes between two scenarios: one where allied attackers confront a single defender, and another where a single attacker faces off against allied defenders. This will involve examining the equilibrium levels of arming allocations in both scenarios.

In terms of the optimal arming decisions, we have from (20a) and (22) the following:

$$G_A^{SLA} - G_A^{MLA} = \frac{V}{8} - \frac{V}{8} = 0 \text{ and } G_C^{SLA} - G_C^{MLA} = \frac{V}{4} - \frac{V}{4} = 0.$$

As for the difference in the overall intensity of conflict, we have from (20b) and (23) that:

$$CI^{SLA} - CI^{MLA} = \frac{V}{2} - \frac{V}{2} = 0.$$

According to (21a). (21b), and (24), the differences in the expected payoffs of the players are:

$$\pi_A^{SLA} - \pi_A^{MLA} = \frac{V}{8}(4\alpha - 1) - \frac{V}{8}(4\alpha - 1) = 0,$$

$$\pi_B^{SLA} - \pi_B^{MLA} = \frac{V}{8}(3 - 4\alpha) - \frac{V}{8}(3 - 4\alpha) = 0,$$

$$\pi_C^{SLA} - \pi_C^{MLA} = \frac{V}{4} - \frac{V}{4} = 0.$$

The implications of these results are summarized in the following proposition:

PROPOSITION 4. In the three-player sequential contests, the equilibrium results for the expected payoffs, arming allocations, and conflict intensity are the same when allied players move first in attacking a stand-alone player that moves second as a defender or when a stand-alone player moves first in attacking allied players that move second as defenders. Furthermore, moving first for a lone player as an attacker does not provide a strategic advantage against allied players that move second as defenders. Likewise, allied players moving first as attackers do not provide a strategic advantage against a lone player that moves second as a defender.

Proposition 4 indicates that the formation of an alliance, whether by players that move first as attackers or second as defenders, neutralizes the strategic advantage of the first mover(s) in the sequential game. In other words, being in a leadership position, whether for multiple players as allied attackers or a stand-alone player as an attacker, does not provide any significant strategic advantage against allied opponents.

The case of the Arab states launching offensive strategic attacks on Israel in 1967 with the goal of penetrating deep into its territory ultimately ended in failure. This historical example aligns closely with our finding that when multiple players form an alliance and engage in an offensive attack against a single defender, such an alliance does not provide a strategic advantage.

5. Alliance vs. non-alliance in sequential-move contests

This section compares the equilibrium outcomes when multiple players ally to make their arming decisions jointly to the alternative scenario when their arming decisions are made autonomously. First, we look at how multiple players that move first as attackers affect arming allocations, conflict intensity and the expected payoffs. Second, we examine the case when the multiple players move second as defenders.

5.1. Alliance vs. non-alliance players attacking a stand-alone defender

Recall that "*MLN*" stands for multiple attackers with no alliance and "*MLA*" stands for multiple attackers with an alliance. We have from (4) and (20a) the following results:

$$G_j^{MLN} - G_j^{MLA} = \frac{9V}{32} - \frac{V}{8} = \frac{5V}{32} > 0 \text{ for } j = A, B, \text{ and } G_C^{MLN} - G_C^{MLA} = \frac{3V}{16} - \frac{V}{4} = -\frac{V}{16} < 0.$$

As for the differences in the overall conflict intensity, we have from (5) and (20b) that:

$$CI^{MLN} - CI^{MLA} = \frac{3V}{4} - \frac{V}{2} = \frac{V}{4} > 0.$$

According to (6), (21a), and (21b), the differences in the expected payoffs are:

$$\pi_A^{MLN} - \pi_A^{MLA} = \frac{3V}{32} - \frac{V}{8}(4\alpha - 1) = \frac{V}{32}(7 - 16\alpha),$$

$$\pi_B^{MLN} - \pi_B^{MLA} = \frac{3V}{32} - \frac{V}{8}(3 - 4\alpha) = \frac{V}{32}(16\alpha - 9),$$

$$\pi_C^{MLN} - \pi_C^{MLA} = \frac{V}{16} - \frac{V}{4} = -\frac{3V}{16} < 0.$$

It is easy to verify that:

$$\pi_A^{MLA} > \pi_A^{MLN} \text{ and } \pi_B^{MLA} > \pi_B^{MLN} \text{ when } \frac{7}{16} < \alpha < \frac{9}{16}.$$

The implications of these results are summarized in the following proposition:

PROPOSITION 5. In a sequential-move game with multiple first-movers attacking a lone player that is a second-mover in defense, we have: (i) Conflict intensity is lower when the multiple attackers jointly make their arming decisions in fighting against a single defender than when the multiple attackers make their arming decision non-cooperatively; (ii) The expected payoffs of the allied attackers and the single defender are higher than those when the attackers make their arming decision non-cooperatively. That is,

(*i*) $CI^{MLA} < CI^{MLN}$,

(ii) $\pi_A^{MLA} > \pi_A^{MLN}$ and $\pi_B^{MLA} > \pi_B^{MLN}$when $\frac{7}{16} < \alpha < \frac{9}{16}$; $\pi_C^{MLA} > \pi_C^{MLN}$.

Proposition 5 implies that a sequential contest with multiple players moving first as attackers while making their arming decisions autonomously against a lone player moving second as a defender is *conflict-aggravating*. In contrast, a sequential contest with multiple players moving first as attackers in fighting against the lone player as a defender is *conflict-reducing*.

While the expected payoffs are relatively higher when multiple players arm collectively as attackers against a single defender and agree on a share of αsuch that $\frac{7}{16} < \alpha < \frac{9}{16}$, the two allied players do not have the strategic advantage in winning against the single defender. When $\alpha > \frac{9}{16}$ or $\alpha < \frac{7}{16}$ such that one player is requesting a relatively high share $\left(\alpha > \frac{9}{16}\right)$, the other player does not have an incentive to form an alliance. In this case, multiple players that arm independently as attackers earn a lower expected payoff, but they have a strategic advantage in winning against the single defender.

5.2. A lone player moves first, attacking either allied or non-allied players that move second

Recall that "*SLN*" stands for the case of a single attacker/leader with no alliance and that "*SLA*" stands for that of a single attacker/leader with an alliance. We have from (10) and (22) the following results:

$$G_j^{SLN} - G_j^{SLA} = \frac{3V}{16} - \frac{V}{8} = \frac{V}{16} > 0 \text{ for } j = A, B, \text{ and } G_C^{SLN} - G_C^{SLA} = \frac{3V}{8} - \frac{V}{4} = \frac{V}{8} > 0.$$

As for the difference in the overall conflict intensity, it follows from (11) and (23) that:

$$CI^{SLN} - CI^{SLA} = \frac{3V}{4} - \frac{V}{2} = \frac{V}{4} > 0.$$

According to the results in (12) and (24), the differences in the expected payoffs are:

$$\pi_A^{MLN} - \pi_A^{MLA} = \frac{3V}{32} - \frac{V}{8}(4\alpha - 1) = \frac{V}{32}(7 - 16\alpha),$$

$$\pi_B^{MLN} - \pi_B^{MLA} = \frac{3V}{32} - \frac{V}{8}(3 - 4\alpha) = \frac{V}{32}(16\alpha - 9),$$

$$\pi_C^{SLN} - \pi_C^{SLA} = \frac{V}{8} - \frac{V}{4} = -\frac{V}{8} < 0.$$

It is easy to verify that:
$\pi_A^{SLA} > \pi_A^{SLN}$ and $\pi_B^{SLA} > \pi_B^{SLN}$ when $\frac{3}{8} < \alpha < \frac{5}{8}$.
The implications of these results are summarized in the following proposition:

PROPOSITION 6. In a sequential-move game with a single attacker and multiple defenders, we have: (i) Conflict intensity is lower when the multiple arming-cooperative defenders fight against the single attacker than when the multiple arming-independent defenders; (ii) The expected payoffs of the multiple arming-cooperative defenders and the single attacker are higher than those when the multiple defenders make their arming decision non-cooperatively. That is,

(i) $CI^{SLA} < CI^{SLN}$,

(ii) $\pi_A^{SLA} > \pi_A^{SLN}$ and $\pi_B^{SLA} > \pi_B^{SLN}$ for $\frac{3}{8} < \alpha < \frac{5}{8}$; $\pi_C^{SLA} > \pi_C^{SLN}$.

Proposition 6 indicates that an alliance between multiple players that move second as defenders is *conflict-reducing,* while a stand-alone player that moves first as attacker in fighting against multiple arming-independent or non-alliance defenders is *conflict-aggravating.* Alliance formations have deterrent effects on the lone player moving first as an attacker.

The expected payoffs are relatively higher for multiple players that make arming decisions jointly as defenders against a single attacker when they agree upon a share of $\alpha$where $\frac{3}{8} < \alpha < \frac{5}{8}$. In this case, the allied players' defensive strategy is effective in deterring the single attacker. When $\alpha > \frac{5}{8}$or$\alpha < \frac{3}{8}$, instead, one player requests relatively high share so that the other player does not have an incentive to form an alliance. In this case, multiple players that make their arming decisions non-cooperatively as defenders fail to deter the single attacker.

Propositions 5 and 6 demonstrate that an alliance may have a pacifying effect in reducing the intensity of conflict. In Proposition 5, an alliance formed by multiple attackers causes them to lose their strategic advantage, while an alliance by multiple defenders deters a single attacker from gaining a strategic edge. Our findings suggest that an offensive strategy is effective when multiple attackers make their arming decisions independently. Conversely, a defensive strategy proves effective when multiple defenders make their arming decisions jointly.

6. Concluding remarks

Researchers looking into market competition and non-market rivalry have paid a great deal of attention to the roles that alliances play and the circumstances in which players have a strategic edge over their opponents. The moving order in a sequential-move game may strategically affect the equilibrium outcomes of a two-player contest, as has been stated in the literature.

In this paper, we examine how alliance formation and moving order interact in affecting the allocations of fighting efforts, the overall intensity of conflict, and each player's strategic advantage in three-player sequential-move contests. We find that multiple players acting as first movers have a strategic advantage over a stand-alone defender only when they have discretion to make arm decisions autonomously or independently. If multiple attackers form an alliance and make their arming decisions jointly, they end up losing the strategic advantage. That is, alliance reduces each alliance player's incentive for arming. Furthermore, a single player moving first as an attacker may have a strategic advantage in fighting against multiple players moving second as defenders when they make their arming decisions non-cooperatively. Third, compared to the equilibrium outcome in a simultaneous-move game, the overall conflict intensity is higher if multiple players arm independently and is lower if they arm cooperatively in an alliance. The three players' expected payoffs are the highest when the multiple players form an alliance, irrespective of whether they are first or second movers.

This paper contributes to the theoretical conflict literature by stressing the different implications between defensive and offensive strategies. The limitations of the model and, hence, potentially interesting extensions should be mentioned. One valuable direction is to allow for asymmetric information regarding players involved in conflict, such as valuation or military capabilities. Another extension is to endogenize the choice of alliance formation rather than taking alliances as exogenously given. Analyzing the emergence of an effective alliance based on the strategic environment could render additional insights for explaining the equilibrium outcomes of conflicts. Further understanding and implication can be derived if we consider asymmetric valuations, the effectiveness of armament technology, and the marginal cost of arming. Our analysis abstracts from resource transfer between allied members, which could affect players' incentives to join an offensive or a defensive alliance, strategic positioning, and conflict outcomes.

Incorporating a scenario that involves more than three players would introduce complexity to our model and affect conflict outcomes, such as conflict intensity, expected payoffs, and the strategic advantage of the first mover(s) and the second mover(s) defensive strategy. The first-mover advantage identified in Proposition 1 may be altered when a fourth player joins the stand-alone defender. Conflict intensity could become higher or lower as potential alliances increase or decrease. Moreover, as the number of players increases, each player's incentive to join an alliance or arm independently depends on the potential expected payoff. While the analysis of such a model with multi-player scenarios is beyond our paper's scope, a future extension could provide valuable insights into understanding the dynamics of alliances in a world with more complex conflicts involving multiple states.

Declaration of competing interest

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors. Nor do the authors have any financial and personal relationships with other organizations or people that could inappropriately influence or bias their work.

Appendix

A-1 *Benchmark equilibrium levels of arming allocations*

The first-order conditions (FOCs) for the three players are given, respectively, as follows:

$$\frac{\partial \pi_A}{\partial G_A} = \frac{\partial}{\partial G_A}\left(\frac{G_A}{G_A + G_B + G_C} V - G_A\right) = \frac{(G_B + G_C)V}{(G_A + G_B + G_C)^2} - 1 = 0, \tag{a.1}$$

$$\frac{\partial \pi_B}{\partial G_B} = \frac{\partial}{\partial G_B}\left(\frac{G_B}{G_A + G_B + G_C} V - G_B\right) = \frac{(G_A + G_C)V}{(G_A + G_B + G_C)^2} - 1 = 0, \tag{a.2}$$

$$\frac{\partial \pi_C}{\partial G_C} = \frac{\partial}{\partial G_C}\left(\frac{G_C}{G_A + G_B + G_C} V - G_B\right) = \frac{(G_A + G_B)V}{(G_A + G_B + G_C)^2} - 1 = 0. \tag{a.3}$$

Without imposing symmetry, the FOCs in (a.1)-(a.3) imply that $G_A = G_B = G_C$.It follows that:
$(G_A + G_A)V = (G_A + G_A + G_A)^2$ or $2G_A V = 9{G_A}^2$.
We thus have $G_A = G_B = G_C = \frac{2V}{9}$.

A-2 *Arming-independent players move first in attacking a stand-alone player moving second*

For the analysis in Section 3.1, we use backward induction and begin with the second stage at which player C determines its arming decision. As in (a.2), the FOC for player C is:

$$\frac{\partial \pi_C}{\partial G_C} = \frac{\partial}{\partial G_C}\left(\frac{G_C}{G_A + G_B + G_C} V - G_B\right) = \frac{(G_A + G_B)V}{(G_A + G_B + G_C)^2} - 1 = 0.$$

Solving for player C's arming allocation as a reaction function of arming allocations by players A and B yields

$$G_C = \sqrt{(G_A + G_B)V} - G_A - G_B. \tag{a.4}$$

Plugging G_C from (a.4) back into the payoff functions of players A and B, we have

$$\pi_A = \frac{G_A}{G_A + G_B + \sqrt{(G_A + G_B)V} - G_A - G_B} V - G_A,$$

$$\pi_B = \frac{G_B}{G_A + G_B + \sqrt{(G_A + G_B)V} - G_A - G_B} V - G_B.$$

The FOCs for players A and B are given, respectively, as follows:
$\frac{\partial \pi_A}{\partial G_A} = \frac{(G_A + 2G_B)}{(G_A + G_B)^{\frac{3}{2}}} \frac{\sqrt{V}}{2} - 1 = 0$ and $\frac{\partial \pi_B}{\partial G_B} = \frac{(G_B + 2G_A)}{(G_A + G_B)^{\frac{3}{2}}} \frac{\sqrt{V}}{2} - 1 = 0$.
Without imposing symmetry, these FOCs imply that:

$$G_A^{MLN} = G_B^{MLN} = \frac{9V}{32}. \tag{a.5}$$

Substituting G_A^{MLN} and G_B^{MLN} from (a.5) back into G_C in (a.4), we have player C's arming allocation:

$$G_C^{MLN} = \sqrt{(G_A^{MLN} + G_B^{MLN})V} - G_A^{MLN} - G_B^{MLN} = \sqrt{\left(\frac{9V}{32} + \frac{9V}{32}\right)V} - \frac{9V}{32} - \frac{9V}{32} = \frac{3V}{16}$$

A-3 *A lone player moves first in attacking multiple arming-independent defenders*

For the analysis in Section 3.2, we use backward induction and begin with the second stage at which players A and B determine their arming decisions independently. As in (a.1) and (a.2), the FOCs for players A and B are given, respectively, as follows:

$$\frac{\partial \pi_A}{\partial G_A} = \frac{\partial}{\partial G_A}\left(\frac{G_A}{G_A + G_B + G_C} V - G_A\right) = \frac{(G_B + G_C)V}{(G_A + G_B + G_C)^2} - 1 = 0,$$

$$\frac{\partial \pi_B}{\partial G_B} = \frac{\partial}{\partial G_B}\left(\frac{G_B}{G_A + G_B + G_C} V - G_B\right) = \frac{(G_A + G_C)V}{(G_A + G_B + G_C)^2} - 1 = 0,$$

Solving for G_A and $G_B$yields:

$$G_A = G_B = \frac{V}{8} - \frac{G_C}{2} + \frac{[(V + 8G_C)V]^{\frac{1}{2}}}{8}$$

Plugging the above results into the payoff function of player C, we have:

$$\pi_C = \frac{[(V + 8G_C)V]^{\frac{1}{2}}}{2} - \frac{V}{2} - G_C$$

The FOC for player C is:

$$\frac{\partial \pi_C}{\partial G_C} = \frac{2}{\sqrt{1+\frac{8G_C}{V}}} - 1 = 0.$$

Solving for the optimal arming allocations for player C yields:

$$G_C^{SL} = \frac{3V}{8}.$$

Plugging G_C^{SL} into the arming reaction functions of players A and B, we have: :

$$G_A^{SL} = G_B^{SL}$$
$$= \frac{1}{8}V - \frac{1}{2}G_C^{SL} + \frac{1}{8}V\sqrt{\frac{1}{V}\left(V + 8G_C^{SL}\right)}$$
$$= \frac{1}{8}V - \frac{1}{2}\left(\frac{3}{8}V\right) + \frac{1}{8}V\sqrt{\frac{1}{V}\left(V + 8\left(\frac{3}{8}V\right)\right)} = \frac{3}{16}V$$

$$G_C^{SL} - G_A^{SL} = \frac{3V}{8} - \frac{3V}{16} = \frac{3V}{16} > 0$$

A-**4** *Allied players move first in attacking a lone player that moves second as a defender*

For the analysis of Section 4.1, we use backward induction and begin with the second stage at which player C makes its arming decision. As in (a.3), the player C's FOC is:

$$\frac{\partial \pi_C}{\partial G_C} = \frac{\partial}{\partial G_C}\left(\frac{G_C}{G_A+G_B+G_C}V - G_c\right) = \frac{(G_A+G_B)V}{(G_A+G_B+G_C)^2} - 1 = 0.$$

Solving for player C's arming allocation yields

$$G_C = \sqrt{(G_A+G_B)V} - G_A - G_B.$$

Note that the joint payoff function of players A and B is:

$$\Pi = \pi_A + \pi_B = \frac{(G_A+G_B)}{(G_A+G_B)+G_C}V - G_A - G_B.$$

Plugging G_C into the above joint payoff function yields

$$\Pi = \pi_A + \pi_B = \frac{(G_A+G_B)}{\left(G_A+G_B+\sqrt{V(G_A+G_B)} - (G_A+G_B)\right)}V - G_A - G_B.$$

The FOCs for the allied players in jointly determining their arming allocations are:

$$\frac{\partial \Pi}{\partial G_A} = \frac{\partial \Pi}{\partial G_B} = -\frac{1}{2(G_A+G_B)}\left(2G_A + 2G_B - \sqrt{V(G_A+G_B)}\right) = 0$$

The FOCs imply that:

$$G_A^{MLA} = \frac{V}{8} \text{ and } G_B^{MLA} = \frac{V}{8}.$$

Substituting G_A^{MLA} and G_B^{MLA} back into G_C, we have player C's arming allocation:

$$G_C^{MLA} = \sqrt{\left(\frac{V}{8}+\frac{V}{8}\right)V} - \frac{V}{8} - \frac{V}{8} = \frac{V}{4}.$$

A-5 *A lone player moves first in attacking multiple arming-cooperative defenders*

For the analysis in Section 4.2, we begin with the second stage at which players A and B make their arming decisions jointly to maximize the sum of their expected payoffs.

$$\Pi = \pi_A + \pi_B = \frac{(G_A+G_B)}{(G_A+G_B)+G_C}V - G_A - G_B.$$

The FOCs for the allied players in jointly determining their arming allocations are:

$$\frac{\partial \Pi}{\partial G_A} = \frac{\partial \Pi}{\partial G_B} = -\frac{1}{(G_A+G_B+G_C)^2}\left((G_A+G_B)^2 + G_C(2G_A+2G_B+G_C) - VG_C\right) = 0$$

The FOCs imply that:

$$G_A = G_B = \left(\frac{\sqrt{VG_C}}{2} - \frac{G_C}{2}\right)$$

Plugging the above result into player C's payoff function, we have

$$\pi_C = \left(\frac{G_C}{\sqrt{VG_C}}\right)V - G_C.$$

The FOC for player C implies that:

$$\frac{\partial \pi_C}{\partial G_C} = -\frac{1}{2G_C}\left(2G_C - \sqrt{VG_C}\right) = 0.$$

Solving for player C's arming allocation yields

$$G_C^{SLA} = \frac{V}{4}.$$

Plugging G_C^{SLA} back into the arming allocations of players A and B, we have

$$G_A^{SLA} = G_B^{SLA} = \frac{1}{2}\sqrt{V\left(\frac{V}{4}\right)} - \frac{1}{2}\left(\frac{V}{4}\right) = \frac{V}{8}.$$

$\blacksquare$

Data availability

No data was used for the research described in the article.

References

Aanesen, M., 2011. Sequential bargaining, external effects of agreement, and public intervention. J. Econ. 105, 145–160.
vol. 3 Bagwell, K., Wolinsky, A., 2002. Game theory and industrial organization. In: Aumann, R, Hart, S (Eds.), Handbook of Game Theory. North Holland. Barclay, Michael, E, New York. vol. 31851-196.
Baik, K.H., Shogren, J.F., 1992. Strategic behavior in contests: comment. Am. Econ. Rev. 82, 359–362.
Beard, S., Strayhorn, J.A., 2018. When will states strike first? Battlefield advantages and rationalist war. Int. Stud. Q. 62, 42–53.
Boudreau, J.W., Sanders, S., Shunda, N., 2019. The role of noise in alliance formation and collusion in conflicts. Public Choice 179, 249–266.
Chang, Y.M., Luo, Z., Zhang, Y., 2018. The timing of third-party intervention in social conflict. Def. Peace Econ. 29, 91–110.
Chang, Y.M., Potter, J., Sanders, S., 2007. War and peace: third-party intervention in conflict. Eur. J. Polit. Econ. 23, 954–974.
Chang, Y.M., Sanders, S., 2009. Raising the cost of rebellion: the role of third-party intervention in intra-state conflict. Def. Peace Econ. 20, 149–169.
Chang, Y.M., Sellak, M., 2022. A theory of competing interventions by external powers in intrastate conflicts: implications for war and armed peace. Appl. Econ. 54, 3811–3822.
Charap, S., Treyger, E., Geist, E., 2019. Understanding Russia's Intervention in Syria. RAND Corporation, Washington, DC.
Congleton, R.D., Hillman, A.L., Konrad, K.A., 2008. 40 Years of Research on Rent Seeking (two volumes). Springer, New York.
Corchón, L.C., Marini, M.A., 2018. Handbook of Game Theory and Industrial Organization, Volume ii: Applications. Edward Elgar Publishing.
Dixit, A., 1987. Strategic behavior in contests. Am. Econ. Rev. 77, 891–898.
Eichenberg, R., 2005. Victory has many friends: U.S. public opinion and the use of military force, 1981–2005. Int. Secur. 30, 140–177.
Gershenson, D., Grossman, H.I., 2000. Civil conflict: ended or never ending? J. Conflict Resolut. 44, 807–821.
Gortzak, Y., Haftel, Y.Z., Sweeney, K., 2005. Offense-defense theory: an empirical assessment. J. Conflict Resolut. 49, 67–89.
Hausken, K., Bier, V.M., 2011. Defending against multiple different attackers. Eur. J. Oper. Res. 211, 370–384.
Jian, L., Li, Z., Liu, T.X., 2017. Simultaneous versus sequential all-pay auctions: an experimental study. Exp. Econ. 20, 648–669.
Jost, P.J., Krakel, M., 2005. Preemptive behavior in sequential-move tournaments with heterogeneous agents. Econ. Gov. 6, 245–252.
Ke, C., Konrad, K.A., Morath, F., 2013. Brothers in arms - an experiment on the alliance puzzle. Games Econ. Behav. 77, 61–76.
Ke, C., Konrad, K.A., Morath, F., 2015. Alliances in the shadow of conflict. Econ. Inq. 53, 854–871.
Konrad, K.A., Leininger, W., 2007. The generalized Stackelberg equilibrium of the all-pay auction with complete information. Rev. Econ. Des. 11, 165–174.
Leininger, W., 1993. More efficient rent-seeking – a munchhausen solution. Public Choice 75, 43–62.
Linster, B.G., 1993. Stackelberg rent-seeking. Public Choice 77, 307–321.
Morgan, J., 2003. Sequential contests. Public Choice 116, 1–18.
Nelson, A.B., 2020. Deterrence in sequential contests: an experimental study. J. Behav. Exp. Econ. 86, 101541.
R. Pukhov (2017). "Russia's unexpected military victory in Syria," Defense News, December 10.
Regan, P., 1996. Conditions for successful third party intervention in intrastate conflicts. J. Conflict Resolut. 40, 336–359.
Regan, P., 2002. Third-party interventions and the duration of intrastate conflicts. J. Conflict Resolut. 46, 55–73.
Roberts, J., 2014. Offense, defense, and the causes of civil war recurrence: the effect of external military support on peace duration. Hinckley J. Politics 15, 28–37.
Rowlands, D., Carment, D., 2006. Force and bias: towards a predictive model of effective third party intervention. Def. Peace Econ. 17, 435–456.
Shinkai, T., 2000. Second mover disadvantages in a three-player Stackelberg game with private information. J. Econ. Theory 90, 293–304.
Siqueira, K., 2003. Conflict and third-party intervention. Def. Peace Econ. 14, 389–400.
Tullock, G., 1980. Efficient rent seeking. In: Buchanan, J., Tollison, R., Tullock, G. (Eds.), Toward a Theory of Rent-Seeking Society. A&M University Press, College Station, pp. 97–112.
Van Evera, S., 1998. Offense, defense, and the causes of war. Int. Secur. 22, 5–43.